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The following Subjects are required for the University of London.

Mechanics.

The Composition & Resolution of Forces.

The Mechanical Powers.

The Centre of Gravity.

The general laws of Motion. (Dynamics)

The Motion of Falling Bodies in free space & down inclined planes.

[Snowball, John Charles]

THE

CAMBRIDGE COURSE

OF

ELEMENTARY NATURAL PHILOSOPHY;

BEING THE DEMONSTRATIONS OF THE PROPOSITIONS IN

MECHANICS AND HYDROSTATICS

IN WHICH THOSE PERSONS WHO ARE NOT CANDIDATES FOR HONOURS ARE EXAMINED FOR THE DEGREE OF B.A.

THE THIRD EDITION.

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ADVERTISEMENT.

THE subjects in which those persons who are not Candidates for Honours are examined for the degree of B.A. are,

I. The Acts of the Apostles in the original Greek.

II. One Greek Classic; appointed in the last week of the Lent Term in every year for the year next but one following. Thus the Classical subjects for the Examinations of 1844 were apppointed in the last week of the Lent Term of 1842.

III. One Latin Classic; appointed in the same manner as the last subject.

IV. Paley's Moral Philosophy.

V. Such Mathematical Subjects as are contained in the following Schedule*.

* The Examiners are required to publish the names of the persons whom their respective Colleges permit to offer themselves for Examination at the Bachelors' Commencement, in a list, which is to be arranged in alphabetical order, and separated into two Divisions. The distribution of the different subjects, and the times of Examination, are fixed by the following Table.

	Div.	Hours of Examination. 9 to 12.	Div.	$12\frac{1}{2}$ to $3\frac{1}{2}$.
Wednesday ..	1	Acts of the Apostles	2	Greek Subject.
Thursday	1	Greek Subject	2	Acts of the Apostles.
Friday	1	Euclid.....	2	Latin Subject.
Saturday	1	Latin Subject.....	2	Euclid.
Monday	1	Mechanics and Hydrostatics	2	Moral Philosophy.
Tuesday	1	Moral Philosophy.....	2	Mechanics and Hydrostatics.
Wednesday ..	1	Arithmetic and Algebra..	2	Arithmetic and Algebra.

Schedule of Mathematical Subjects of Examination, for the Degree of B.A., of Persons not Candidates for Honours.

Arithmetic.

Addition, subtraction, multiplication, division, reduction, rule of three; the same rules in vulgar and decimal fractions; practice, simple and compound interest, discount, extraction of square and cube roots, duodecimals.

Algebra.

1. Definitions and explanation of algebraical signs and terms.
2. Addition, subtraction, multiplication, and division of simple algebraical quantities and simple algebraical fractions.
3. Algebraical definitions of ratio and proportion.
4. If $a : b :: c : d$ then $ad = bc$, and the converse:
   also $b : a :: d : c$,
   and $a : c :: b : d$,
   and $a+b : b :: c+d : d$.
5. If $a : b :: c : d$,
   and $c : d :: e : f$,
   then $a : b :: e : f$.
6. If $a : b :: c : d$,
   and $b : e :: d : f$,
   then $a : e :: c : f$.
7. Geometrical definition of proportion. (Euclid. Book v. Def. 5.)
8. If quantities be proportional according to the algebraical definition they are proportional according to the geometrical definition.
9. Definition of a quantity *varying as* another, *directly* or *inversely*, or as two others *jointly*.

Euclid.

Books i. ii. iii.;

Book vi. Props. 1. 2. 3. 4. 5. 6.

[The remaining portion of the Schedule forms the Table of Contents of this Book.]

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ARTS.

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18. Prop. 10. To describe the *hydrostatic balance*, and to shew how to find the specific gravity of a body by means of it, 1st, when its specific gravity is greater than that of the fluid in which it is weighed; 2dly, when it is less.

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THE ELEMENTS

OF

MECHANICS AND HYDROSTATICS.

The explanatory articles and paragraphs which are enclosed in brackets, form no part of the Course adopted by the University. Those in smaller type are illustrative of the Definitions and Propositions they immediately follow. The whole of the extra matter so introduced forms but a very small, and a very easy, portion of the book; and the student who comes to this subject for the first time had better not omit it.

THE ELEMENTS OF MECHANICS.

CHAPTER I.

DEFINITIONS; EXPLANATION OF STATICAL FORCES; THE MANNER IN WHICH THEY ARE MEASURED, AND REPRESENTED.

1. The science of Mechanics investigates the causes that prevent or produce motion in bodies, or that tend to produce or prevent motion.

It is divided into two parts. The one, which investigates the conditions fulfilled when a body is in a state of rest, is called Statics. The other, which treats of the causes and the effects of motion, is called Dynamics.

Thus it is the province of Statics to shew how the roof of a building is supported by the cross beams and the walls. If the roof gave way and fell, it would belong to Dynamics to account for the circumstances attending the fall; to explain why the motion took place in one direction rather than in another, to determine the time

elapsed in falling, and the swiftness of motion at any instant.

The part of MECHANICS treated on in the following pages is THE ELEMENTS OF STATICS.

2. DEF. Whatever the cause may be that produces or prevents motion, or that tends to produce or prevent motion, it is called a FORCE. Read Page 15. 7.

[If a heavy body, as a stone, be laid on the open hand, experience shews that to prevent the stone from falling the hand must make some effort. Again, to set a ball rolling along the ground requires some exertion. The effort, or exertion, is called in either case a FORCE: and although the effect produced be not great enough to prevent entirely the fall of the stone, or to communicate motion to the ball, yet it is still denominated a FORCE.

From the definition of STATICS given in Art. 1. it will readily be understood, that in that branch of MECHANICS the conditions are investigated which are fulfilled by those forces only which keep a body at rest.]

3. DEFS. (1) All bodies, if left to themselves, would fall to the earth's surface. This power that resides in the earth, of drawing all substances to its surface, is called THE FORCE OF GRAVITY.

(2) The act of a body's pressing downwards to the earth when laid on any substance is called the GRAVITATION of the body.

(3) The *precise amount* of gravitation residing in any particular body is termed the WEIGHT of that body.

Read Page 14. 3.

4. [*The* WEIGHTS *of different bodies may be thus compared.* If two bodies, when successively attached in the same manner to a spring, so that they may act upon it by their weights in the same way, produce the same effects,—(by bending the spring to the same extent),—the weights of the bodies will be equal. Any other body that produces the same effect on the spring by its weight as both the former bodies when applied at the same time do by their weights, is double the weight of either of them. And by means of such a contrivance as this spring, bodies might be shewn to be three, four, or any number of times the weight of the first body.

The WEIGHT of any body is thus *measured.* The weight of a certain bulk of some particular substance is first fixed upon as a standard. Thus the weight of a piece of *lead* of a certain size being called a *pound*, any other body whose gravitation produces the same effect as four, or six, or ten such pieces of lead, will be four, or six, or ten pounds in weight, as the case may be.]

, Composed

5. DEF. The substance, (material, or stuff,) of which any body is made, is called MATTER. Read Page 14. R.

[As all bodies have Weight, the property of having Weight is to be considered as necessarily and inherently belonging to all Matter; and it is with respect to this its property of having Weight that Matter is made the subject of mathematical investigation. The existence of Matter, under whatever form it may appear, is therefore tested by that form having Weight; and in the same ratio or degree that one body has more Weight than another, it is concluded that it contains more Matter.]

6. DEF. THE QUANTITIES OF MATTER which different bodies contain are proportional to the weights of the bodies. Read Page 15. A.

[Thus, if a body *A* weigh *one* pound, and another body *B* weigh *three* pounds, the quantity of matter contained in *A* is *said* to be to the quantity contained in *B* as 1 to 3; or *B* is said to contain three times as much matter as *A* does.

The exact quantity of matter contained in any body is measured by comparing its weight with the weight of some particular body that has been fixed upon for a standard. Thus if a *cubic inch* of *water* be previously taken as the body to measure the quantities of matter contained in all other bodies by, and the quantity of matter in this cubic inch of water be called *one*, then the quantity of matter in any other body would properly be said to be *five*, if the weight of that body were five times as great as the weight of the cubic inch of water.]

7. DEF. When the DENSITIES of different substances are spoken of, the closeness is referred to with which the matter composing them is packed, as it were, in the substances.

To compare this closeness, let *equal* bulks, or sizes, of two different substances be taken; and suppose the substances to be Water and Lead. If the bulk of *water* which is taken weigh *one* pound, it will be found that the piece of lead of equal size with it will weigh $11\frac{4}{10}$ pounds. There is evidently, therefore, $11\frac{4}{10}$ times as much heavy matter in a piece of lead as there is in an *equal* bulk of water; and this fact is expressed or described by saying, that "The DENSITY of Lead is to the DENSITY of Water, as $11\frac{4}{10}$ is to 1;" or by saying, "The DENSITY of Lead is $11\frac{4}{10}$ times that of Water."

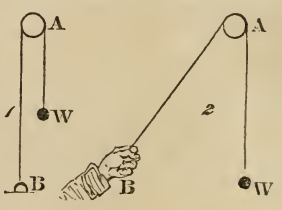
COR. 1. If the density of water be *called* 1, i.e. if *water* be taken to *measure* Density, then the density of lead will be properly called $11\frac{4}{10}$, or 11·4. Read Page 15. T.

[COR. 2. In the same manner as it has been explained how the density of lead is estimated with respect to the density of water, the densities of any other substances, whether solid or fluid, may be determined with respect to that of water.]

8. To explain how the precise amount of a *Statical Force* is described; that is, TO SHEW HOW STATICAL FORCES ARE MEASURED. Read Page 15. 1.

The amount of a Statical Force is described by stating the number of pounds it would support if the Force were made to act directly opposite to the force of gravity. Thus if the weight of a body were P pounds, and it were prevented from moving towards the earth's surface by a hand placed beneath it, the resistance offered by the hand to the communication of motion, (that is, the *force* exercised by the hand), would be P pounds; and if this same pressure were produced by the hand in any other direction, it would be described in the same manner, by saying that it "was equal to P pounds," or that it "was" "P pounds."

9. [FORCES are, in Statics, also called PRESSURES. In whatever direction a force tends to produce motion, its magnitude, (as has already been stated,) is measured by the weight of the body which would exert the same effect to produce motion downwards, as the force under consideration exerts in the line in which *it* endeavours to produce motion. (And that such a method of measuring Forces is allowable appears from *this* consideration,) viz., that the effect produced by the weight of a heavy body may be made to take place in *any* direction whatever; horizontally, as in the case of a string being attached to an object lying on a table and kept stretched by a heavy body (W) that hangs over the side of the table; or vertically upwards, as in fig. 1, by passing the string over a peg A and attaching the end to a ring B, so that BA may be vertical; or in any other direction (fig. 2), by making the heavy body pull the string in a line BA which is inclined at any angle to that (AW) in which it acts itself.



10. (1) The *point* at which a force acts upon a body is called the '*point of application*' of the force.

(2) The *line* in which a force produces, or tends to produce, motion, is called '*the line of the force's action*'; and

Read Page 15. 2

any line that is parallel to the line of a force's action is said to be "*in the direction of the force's action*", or "*in the direction of the force.*" Read Page 15. n

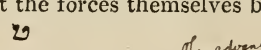
[When the *direction of a force's action*, or as it is generally called "*the direction of the force*," is indicated by a line, either the very line is given in which the force acts, or some line which is parallel to it. "*The line of a force's action*" and "*the direction of the force*" must by no means be confounded together. If the former be known, the latter is necessarily known also; but if only the latter be given, the precise line in which the force produces, or tends to produce, motion, is a matter of uncertainty, and all that can be said respecting it is, that the line of action of the force is either that line, or some other line which is parallel to it.]

(3) If two or more forces be applied to a body, or at some point, and no motion is produced, they are said to "*counteract*," or to "*balance*" one another, or to be "*in equilibrium.*"

11. Forces may be properly represented by lines.

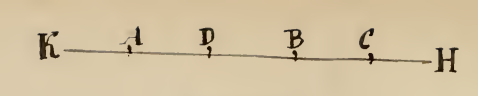
Since lines may be drawn of any length and in any direction from a point, the lines in which forces act, and the ratios which the forces bear to one another, may be represented by drawing lines which coincide with the lines in which the forces act, and whose lengths bear to one another the same ratios that the forces themselves bear to one another. Read Page 15. v

[Among other advantages which attend this method of indicating the magnitudes and directions of forces, the addition and subtraction of such forces as act at a point in the same straight line are easily effected. Thus, if a certain force act at A in the line AH, and AB be taken to represent it, and another force, half the size of the former, act at A in the same line, and also tend to move the body from A towards H, then by taking BC equal to the half of AB the line AC will represent the whole pressure at A, both with respect to the *magnitude* of that pressure and to *the line in which it acts.* And in like manner,



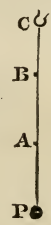
if a force equal to half the original force AB act at A in the line AH, but tends to move the body at A from A towards K, half the pressure of the former force will be counteracted by this new force. Cutting off from the line AB, therefore, a part BD equal to the half of AB, the effective pressure still remaining will be properly represented by AD, both with respect to its *magnitude* and to its *line of action.*]

12. [NOTE. It will be gathered from the last Article, that a force AB applied at A has not the same effect as a force BA applied at that point; for a force AB would



tend to move a body at A in the line KH towards H, but a force BA would tend to move a body at A in the line KH from A towards K. It is not, therefore, indifferent whether the words "a force AB" or "a force BA" be used; since, though the two forces represented by AB and BA be the same in *magnitude*, and also act in the same straight line, yet they tend to produce motions directly opposite to one another; the force AB tending to move the body at A towards H, and the force BA tending to move the body at A towards K.] Read Page 15 ?

13. [The effect produced at a point by any force is the same at whatever point in its line of action the force is applied. Read Page 15 ⊃



For if a heavy body P be suspended by a string CP, the force necessary to prevent P falling to the earth is found to be the same whether that force be applied at A, or B, or C;—the weight of the string being either neglected, or the weight of that portion of it which is supported along with the heavy body P, being counterbalanced.]

14. [DEF. If a string, fastened at one end, be pulled by a force applied at the other end, the resistance to motion made by the string at any point in it is called the TENSION of the string at that point.

If the string be supposed to be without weight, it will follow from Art. 13 that the tension at *every* point of it is the same; namely, the force by which the string is pulled.]

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15. [To recapitulate the substance of this Chapter.

or rather any body is &c.

(1) MATTER is the substance of which all bodies are composed. It is found, by universal experience, to have a tendency to move towards the earth. Art. 5.

(2) The precise amount of the pressure with which any particular body endeavours to move towards the earth, is called the WEIGHT of that body.

This WEIGHT of a body is *measured* by comparing the tendency of the body to move towards the earth with that of some other given body, (of a certain size and formed of a certain material), which is taken for a standard, and to which the name of a *grain*, an *ounce*, a *pound*, or a *ton* is given, as the case may be. Art. 3.

ג (3) THE QUANTITIES OF MATTER contained in different bodies (whether the bodies be great or small, rare like gas or dense like lead), is *proportional*, (not *equal*), to the weights of the bodies. Art. 6.

ד (4) The DENSITIES of different substances are *proportional*, (not *equal*), to the weights of *equal bulks* of the substances. Art. 7.

ה (5) I. Whatever be the cause which moves, or tends to move, matter existing under any form whatsoever, it is called FORCE. Art. 2.

ו FORCES which prevent motion taking place,—that is, STATICAL FORCES,—are measured by the number of *pounds* they would support if they acted vertically upwards. Arts. 2, 8.

ז II. THE LINE OF A FORCE'S ACTION is the actual line in which the force tends to produce, or to prevent, motion. Art. 10.

ח III. THE DIRECTION OF A FORCE is indicated either by the line of its action, or by any line which is parallel to the line of its action. Art. 10.

ט IV. The magnitudes of forces, and the lines (or directions) in which they act, may be represented by means of straight lines. Art. 11.

י V. If a straight line AB represent a force acting on a material point placed at A, and BA represent another force acting on the same point, the two forces AB and BA are equal, and tend to move the point in the same straight line, but in opposite ways from A. Art. 12.

כ VI. A Statical Force produces the same effect at whatever point in its line of action it is applied. Art. 13

1. The magnitude of the force;
2nd. The point of application;
3rd. The line or direction in which it acts.

VII. To investigate the effects produced by a force there must be given;

1st, The magnitude of the force; which is known if the number of pounds be known which the force would support;

2nd, The point at which the force is applied;

3rd, The line in which it acts, or the direction; for knowing the direction of the force and the point it is applied at, the *line* in which the force acts may be determined by drawing a line through the given point parallel to the given direction.]

CHAPTER II.

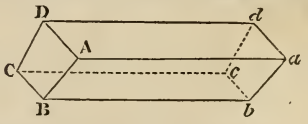
THE LEVER.

16. DEFS. (1) A PLANE, or A PLANE SUPERFICIES, is that superficies, or surface, in which if *any two* points be taken, the straight line joining them lies wholly within the superficies. EUCLID I; Def. 7.

(2) A SOLID is that which hath length, breadth, and thickness. EUCLID XI; Def. 1.

(3) PARALLEL PLANES are such as do not meet one another, however far they may be produced. EUCLID XI; Def. 8.

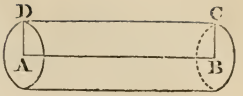
(4) A PRISM is a solid bounded by plane rectilineal figures; two of them, which are parallel to each other, being similar and equal triangles squares or polygons, having the similar angles in each opposite, and all the remaining plane rectilineal figures which bound the prism being rectangles.



[Thus, let $ABCD$ and $abcd$ be two equal and similar quadrilateral figures, placed with their planes parallel, and with their similar angles opposite to each other; and let all the figures, such as $ABba$, which are formed by joining the equal angles of $ABCD$ and $abcd$, be rectangles; then the solid included by these rectangles and by the *ends*, or *bases*, $ABCD$ and $abcd$, is called a *prism*.

The *length* of the prism is any one of the edges Aa, Bb, &c.; which lines, being the sides of adjacent rectangles, are all equal to one another.]

(5) A CYLINDER is a solid described by a rectangle ($ABCD$) revolving round one of its sides (AB) which remains fixed.



[The side AB, which remains fixed, is the *length* of the cylinder, and is called its *axis*.

The surfaces described by the two sides AD and BC of the rectangle which are adjacent to the fixed side, are circles in form, and are parallel to each other; they are called the *ends*, or *bases*, of the cylinder.]

17. DEF. A LEVER is a rigid and inflexible rod, moveable in one plane round a point in the rod called the FULCRUM.

In the following propositions the thickness of the rod is neglected, and the lever is considered to be a geometrical line; it is also supposed to be without weight.

18. The investigation of the properties of the Lever will be made to depend on the following Axioms, which the mind readily admits as true.

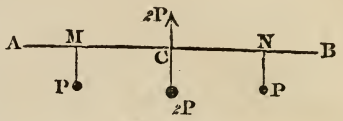
AXIOM I. If two *equal* weights, or forces, act perpendicularly upon a horizontal straight lever, they will balance round a fulcrum placed at the middle point between the points at which they are applied; the pressure on this fulcrum will be the sum of the weights, and it will act in the same direction as the weights act.

AXIOM II. If *any* two weights, or forces, acting perpendicularly on a horizontal straight lever, on opposite sides of a fulcrum, balance each other, the pressure on the fulcrum is equal to the sum of the weights.

AXIOM III. The efforts made by two *equal* forces, acting perpendicularly at the extremities of *equal* straight arms of any lever, to turn the lever round, are equal.

19. Prop. 1. A horizontal prism, or cylinder, of uniform density, will produce the same effect by its weight as if it were collected at its middle point.

Let P and P be two equal weights acting perpendicularly at M and N on the straight horizontal lever AB which is moveable round any fulcrum in it; They produce the same effect as if they were applied at C the middle point between M and N.



At C let a weight $2P$ act perpendicularly downwards, and also let a pressure equal to $2P$ act upwards at C. Since these two forces are equal and opposite they will counteract each other; and whatever the tendency of the lever was to turn round the fulcrum, it will remain unaltered.

Now since P at M and P at N would produce a pressure downwards of $2P$ on a fulcrum at C, (Axiom I.), the pressure upwards exerted by such a fulcrum would be $2P$. The pressure therefore of $2P$, which has been supposed to act upwards at C, will serve instead of such a fulcrum, and will therefore balance P at M and P at N. Wherefore, the three forces, P at M, P at N, and $2P$ acting upwards at C, have no tendency whatever to turn the lever about; but its tendency to turn about, (which has been shewn to be the same as when the lever was acted on by no other forces than P at M and P at N), arises entirely from a force $2P$ applied at C, and acting downwards.

Now supposing* that a horizontal prism (or cylinder) of uniform density, would produce the same effect by its weight, as if the whole matter composing it were collected into a number of small equal weights placed at equal distances along a horizontal straight line of the same length as the prism (or cylinder), then, since it appears from what has been proved above, that *every* pair of weights which are equidistant from the center of the line, would produce the

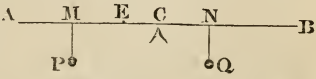
* A uniform prism, or cylinder, may be divided, by means of equally distant planes which are parallel to its ends, into equal small prisms or cylinders lying on each side of the plane passing through its center. Now if these be supposed to be condensed and placed at equal distances along a straight line equal to the length of the original prism, or cylinder, the supposition made in the text will not appear unreasonable.

same effect as if it were collected at the center, the whole prism therefore, or cylinder, (being made up of such pairs of equal weights), will produce the same effect by its weight as if it were collected at its center. Q. E. D.

Cor. Hence it follows that a horizontal prism, or cylinder, of uniform density, will balance on its middle point; and the pressure on a fulcrum placed (there) will be the weight of the prism, or cylinder.

20. Prop. II. If two weights, acting perpendicularly on a straight lever, on opposite sides of the fulcrum, balance each other, they are inversely as their distances from the fulcrum; and the pressure on the fulcrum is equal to their sum.

Let there be an uniform cylinder whose length is AB, and whose weight is $P+Q$. If it be placed horizontally it will balance on a fulcrum C placed at its middle point (Prop. I. Cor.); and the pressure on C will be the weight of the cylinder, namely $P+Q$.



Let AB be divided in E so that $AE : AB :: P : P+Q$; then, since the cylinder is supposed to be of uniform density, Weight of the part AE of the cylinder : weight of the whole :: $AE : AB$

$$:: P : P+Q, \text{ by construction;}$$

but, Weight of the whole $= P+Q$; $\therefore$ weight of the part $AE = P$; and $\therefore$ weight of $EB = Q$.

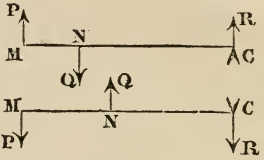
But if the parts AE and EB be collected at their middle points M and N, and act perpendicularly on the lever, they will produce the same effects by their weights as before; as in Prop. I. Therefore, the weights P and Q, acting perpendicularly at M and N, will balance about C, and the pressure on C will be $P+Q$. Now

$$\frac{P}{Q} = \frac{AE}{EB} = \frac{AB-BE}{AB-AE} = \frac{2BC-2BN}{2AC-2AM} = \frac{BC-BN}{AC-AM} = \frac{CN}{CM},$$

$$\therefore \frac{P}{Q} = \frac{CN}{CM}. \quad \text{or } P : Q :: CN : CM.$$

Wherefore, "If two weights, &c." Q. E. D.

21. Prop. III. If two forces, acting perpendicularly on a straight lever in opposite directions and on the same side of the fulcrum, balance each other, (1) they are inversely as their distances from the fulcrum, and (2) the pressure on the fulcrum is equal to their difference.



Let the forces P and Q, acting perpendicularly at M and N on the straight lever MC in opposite directions and on the same side of the fulcrum C, balance each other; Then $P : Q :: CN : CM$, and, (Q being the force which is the nearer to the fulcrum), the pressure on the fulcrum $= Q - P$.

Let the fulcrum at C be removed, and the place of its resistance (R) supplied by a force equal to R, and acting perpendicularly to the lever in the direction of P's action.

Since P and R are counterbalanced by Q, they must produce a pressure at N equal and opposite to Q. Let Q be removed, and its place supplied by a fulcrum on the contrary side of the lever to that on which Q acted, sustaining the pressure (namely Q) produced by P and R.

The case is now that of two forces acting perpendicularly on opposite sides of the fulcrum, and balancing each other; and therefore, by Prop. II,

$$P : R :: CN : NM;$$

$$\therefore P : P + R :: CN : CN + NM :: CN : CM.$$

But Q, the pressure on the fulcrum which has been supposed to be placed at N, is (by Prop. II.) $= P + R$.

$$\therefore P : Q :: CN : CM.$$

Also since $Q = P + R$, $\therefore R = Q - P$.

Wherefore, "If two forces, &c." Q. E. D.

22. [From the last two Propositions it appears, that if a straight lever, which is acted on perpendicularly by two weights, or other forces, P and Q, respectively applied at the distances CM and CN from the fulcrum C, be at rest, then, whether P and Q act on the same or on different sides of the fulcrum, the proportion $P : Q :: CN : CM$ is always true.

When the Lever is used to balance a given weight or force Q, by the application of another force P, Q is usually called "*the Weight*," and P "*the Power*."

If CM, the perpendicular distance from the fulcrum at which the Power acts, be greater than CN, the distance at which the Weight acts, the Power required to balance the Weight is less than it. In this case "*force*" is said "*to be gained*" by the application of the lever. But if CM be less than CN, the Power required to balance the Weight is greater than it, and "*force*" is then said "*to be lost.*"]

23. PROP. IV. To explain the different kinds of levers.

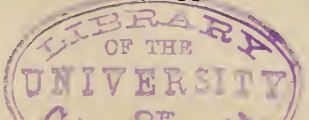
LEVERS are divided into three classes.

(1) Where the Power and the Weight act on *opposite* sides of the fulcrum.

(2) Where the Power and the Weight act on the *same* side of the fulcrum, but the perpendicular distance from the fulcrum at which the Power acts is *greater* than that at which the Weight acts.

(3) Where the Power and the Weight act on the *same* side of the fulcrum, but the perpendicular distance from the fulcrum at which the Power acts is *less* than that at which the Weight acts.

Of the FIRST class the poker, when used to raise the coals, is an instance; the bar of the grate on which the poker rests being the Fulcrum, the force exerted by the hand the Power, and the resistance of the coals the Weight. In the common balance, the Power and the Weight are equal forces perpendicularly applied at the ends of equal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arms. In the steelyard, the Power and the Weight are perpendicularly applied at the ends of unequal arms. Pincers, scissars, and snuffers, are double levers of this kind, the rivet being the fulcrum in each.

Since *CM* may be either greater or less than *CN*, the Power in levers of this class may be either less or greater than the Weight, and consequently "Force" may be either "gained" or "lost" by using them.

Of the SECOND class, a cutting blade, such as those used by coopers, moveable round one end, which is fastened by a staple to a block, by means of a handle fixed at the other, is an example. An oar is also such a lever; the Fulcrum being the extremity of the blade, (which remains fixed, or nearly so, during the stroke), the muscular strength and weight of the rower being the Power, and the Weight being the resistance of the water to the motion of the boat, which is counteracted and overcome at the rowlocks. A pair of nut-crackers is a double lever of the second class.

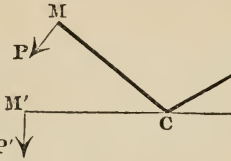
Here, since *CM* is greater than *CN*, the Power is always less than the Weight, or "Force is gained" by using levers of the *second* class.

An example of the THIRD class is the board which the turner (or knifegrinder) presses with his foot to put the wheel of his lathe in motion; the Fulcrum being the end of the board which rests on the ground, the Power being the pressure of the foot, and the Weight being the pressure produced at the crank put on the axletree of the wheel. Fire tongs, and sugar tongs, are double levers of this kind; the Weight in either case being the resistance of the substance grasped. The limbs of animals are also such levers: thus, if a weight be held in the hand and the arm be raised round the elbow as a Fulcrum, the weight is supported by muscles fastened at one extremity to the upper arm, and again attached to the forearm, after passing through a kind of loop at the elbow.

Here, since *CM* is less than *CN*, the Power is greater than the Weight, or "Force is lost" by making use of levers of the *third* class.

24. Prop. v. If two forces acting perpendicularly at the extremities of the (straight) arms of *any* lever, balance each other, they are inversely as the arms.

Let the forces P and Q, acting perpendicularly at the extremities of the straight arms CM and CN of any lever whose fulcrum is C, balance each other; They are inversely as the arms.



Draw the horizontal line $M'N'$ through C, and take $CM' = CM$, and $CN' = CN$. Let a force P', equal to P, act perpendicularly at M'; and a force Q', equal to Q, act perpendicularly at N'.

Now since $CM' = CM$ and $P' = P$, P' will produce the same effect as P on the lever; Axiom III. Art. 18;

But, by supposition, P balances Q;

$\therefore$ P' would balance Q.

And since $CN' = CN$ and $Q' = Q$, Q' will produce the same effect on the lever as Q; Axiom III.

But Q would balance P';

$\therefore$ Q' would balance P';

wherefore, by Prop. II., $P' : Q' :: CN' : CM'$,

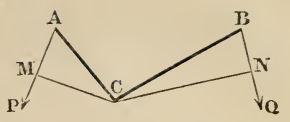
and $\therefore P : Q :: CN : CM$.

Wherefore, "If two forces, &c." Q. E. D.

25. Prop. VI. If two forces acting at any angles on the arms of any lever, balance each other, they are inversely as the perpendiculars drawn from the fulcrum on the directions, (i. e. the lines), in which the forces act.

Let P and Q be two forces which acting at any angles on the arms CA and CB of any lever ACB balance each

other about the fulcrum C; They are inversely as the perpendiculars CM and CN drawn from the fulcrum on the lines in which they act.



Since a force produces the same effect at whatever point in its line of action it is applied (Art. 15), the force P may be supposed to be applied at M; and in order that it *may* be so applied, let a rod (CMA) without weight be fastened to CA. In like manner Q may be supposed to be applied at N perpendicularly to the part CN of the rod CNB which is added to CB.

And, since P acting at M perpendicularly to CM balances Q acting at N perpendicularly to CN; by Prop. v.,

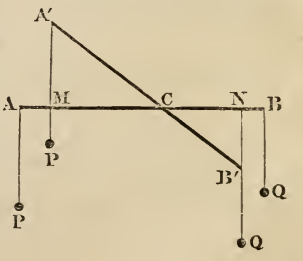
$$P : Q :: CN : CM;$$

and therefore when P and Q acting at A and B in the lines AMP, BNQ balance, they are as $CN : CM$.

Wherefore, "If two forces, &c." Q. E. D.

26. Prop. vii. If two weights balance one another on a straight lever when horizontal, they will balance each other in *every* position of the lever.

Let P and Q be two weights that balance each other round the fulcrum C on the lever ACB when it is horizontal; They will balance each other on the lever when it comes into any other position, as $A'CB'$.



Produce QB' to cut AB in N.

Since weights act perpendicularly to the horizon, $A'P$ and $NB'Q$ are each perpendicular to the horizontal line ACB;

$\therefore$ angle CMA' = right angle = angle CNB',

and angle $A'CM$ = opposite angle $B'CN$; Euc. I. 15.

$\therefore$ also angle $CA'M$ = angle $CB'N$, and the triangles are equiangular, and therefore similar.

Therefore, by Euc. VI. 4,

$$CN : CB' :: CM : CA';$$

$$\therefore CN : CM :: CB' : CA' :: CB : CA.$$

But, $\because$ P and Q balance on ACB, $\therefore CB : CA :: P : Q$;

$$\therefore CN : CM :: P : Q;$$

But CM and CN are the perpendiculars from C on the lines in which P and Q act when they are hung at A' and B'; therefore, by Prop. VI., P and Q will balance on $A'CB'$.

Wherefore, "If two weights, &c." Q. E. D.

CHAPTER III.

THE COMPOSITION AND RESOLUTION OF FORCES.

27. Definition of Component and Resultant Forces.

It is found, by experiment, that a body which is acted on by two forces applied at the same instant to the same point of it, instead of having a tendency to move in either of the lines the forces act in, has a tendency to move in a line lying between them. Whence it appears, that the two original pressures by their combined action produce a third pressure; which pressure is called, from the circumstance of its "resulting," out of the actions of the

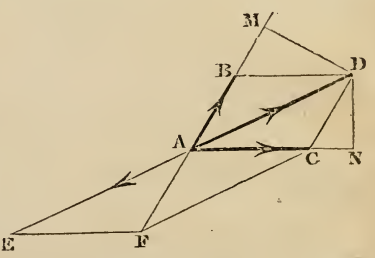
original pressures, their "Resultant" with respect to them; while they are called, with respect to it, its "Components."

The Resultant (R), which produces the same effect as the compound action of the original forces P and Q applied at the same point at the same instant, is said to be "compounded" of P and Q. This Resultant (R) also, if conceived to be the sole original force, may be supposed to be "resolved" into the two forces P and Q; since those two forces, acting in the manner described, (namely, at the same point, and at the same instant), produce exactly the same effect on the body as the single force R does.

[An instance of the "Composition of Forces" is a boat moored in a stream by ropes attached to the boat and to either bank; the Component forces here being the tensions of the ropes, and their Resultant being the force by which the pressure of the stream on the boat is counteracted.]

28. Prop. VIII. If the adjacent sides of a parallelogram represent the component forces [that act at a point at the same instant], in direction [i. e. in their lines of action] and magnitude, the diagonal will represent the resultant force (1) in direction [line of action], and (2) in magnitude.

Let AB and AC represent, in direction and magnitude, the two component forces that act at A. Complete the parallelogram BC, and draw the diagonal AD. Then AD will represent the resultant of AB and AC, (1) in direction, and (2) in magnitude.



From D draw DM and DN perpendicular to AB and AC, produced if necessary.

(1) Then in the triangles DBM, DCN,

angle DMB = right angle = angle DNC,

and angle DBM = angle BAC (since BD, AC are parallel and MBA cuts them), = angle DCN;

$\therefore$ the third angle, BDM, of the one triangle = the third angle, CDN, of the other; and the triangles are equiangular and similar.

$$\therefore CD : DN :: BD : DM,$$

and $\therefore$, alternately, $CD : BD :: DN : DM$.

Now if there be a lever AD whose fulcrum is D, which is acted on by the forces AB, AC applied at A; since

$$\text{Force in the line } AM : \text{force in line } AN :: AB : AC$$
$$:: CD : BD, \text{ (since } BC \text{ is a parallelogram,)}$$
$$:: DN : DM;$$

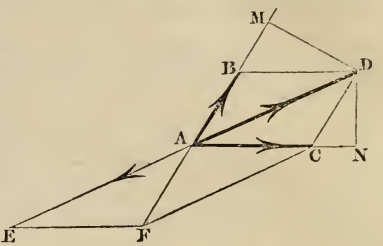
the two forces acting on the lever AD are inversely as the perpendiculars from the fulcrum on their lines of action, and therefore the lever will be kept at rest about D by them (Prop. VI.). Wherefore the lever will also be kept at rest by the Resultant of those forces; because that single force produces the same effect as they do when they act at the same point and at the same instant.

This Resultant therefore *must act in the line AD*, for it keeps the lever at rest, which it could not do were it to act at A and make any angle with the lever AD on either side of it.

(2) Again: Having shewn that the resultant of AB and AC acts in the line of the diagonal, next to prove that the diagonal represents it in *magnitude* as well as in direction.

Produce DA, and suppose a force AE to be taken in it equal and opposite to the Resultant of AB and AC. The joint effect of AB and AC will now be counteracted by AE; and the point A, which is acted on by the three forces AB, AC, and AE, will remain at rest. Whatever be the effect, therefore, produced by the joint action of AE and AC, it is counteracted by AB; that is, AB must be equal and opposite to the Resultant of AE and AC.

Complete the parallelogram EC, and draw the diagonal AF. By the first part of the Proposition, AF is the line in which the resultant of AE and AC acts; and since the force AB is equal and opposite to that resultant, AF must be in the same straight line with AB, and, therefore, it is parallel to CD.

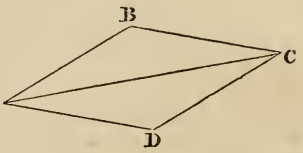


Wherefore, EC and FD are parallelograms; and $AE = FC = AD$. The Resultant, therefore, of AB and AC (which is equal and opposite to AE) will be properly represented in *magnitude* by AD, the diagonal of the parallelogram of which AB and AC are the sides.

Wherefore, "If the adjacent sides, &c." Q. E. D.

29. Prop. IX. If three forces represented in magnitude and direction by the three sides of a triangle [when taken in order], act on a point, they will keep it at rest.

Let the sides AB, BC, CA, taken in order*, of the triangle ABC, represent in magnitude and direction (see Art. 15), three forces that act on the point A; They will keep A at rest.



Complete the parallelogram BD.

Then AD is parallel and equal to BC; and it will, therefore, represent in magnitude and in *line of action* (Art. 15) the force which acts at the point A in the *direction* BC, and is represented in magnitude by BC.

* By the expression "taken in order" is meant, that if ABC be the triangle and AB be one of the forces, BC (and not CB) is the next, and CA (not AC) is the third; so that the forces are described in the same direction round the triangle, proceeding from A to B, from B to C, and from C to A again.

Now the forces AB and AD acting at A will produce a resultant AC. Prop. VIII.

If, therefore, a force CA act at A, the force AC will be counteracted, and the point A will remain at rest; Wherefore, if three forces, represented in magnitude and direction by AB, AD, CA,—(or, which is the same thing, if they be represented by the three sides AB, BC, CA, taken in order, of the triangle ABC)—act on a point A, they will keep it at rest. Q. E. D.

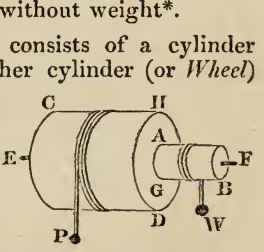
[Cor. It appears from this proof, that if two sides of a triangle ABC, as AB and BC, taken in order, represent in magnitude and direction two forces which act at the same instant at a point A, the third side of the triangle AC, not taken in the same order as AB and BC, represents their Resultant, in magnitude and direction.]

CHAPTER IV.

MECHANICAL POWERS.

30. The Mechanical Powers are certain instruments by which a lesser weight, called the *Power*, may be made to balance a greater, called the *Weight*. The properties will now be investigated of three of them, called the *Wheel and Axle*, the *Pulley*, and the *Inclined Plane*. They will be supposed to be rigid bodies and without weight*.

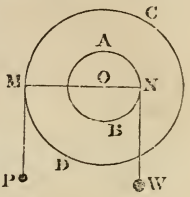
31. The Wheel and Axle consists of a cylinder (or *Axle*) AB, terminating in another cylinder (or *Wheel*) CD of greater base. The two cylinders have a common axis EF, which is supported in a horizontal position. The *Power* (P) is a heavy body which hangs freely by a string coiled round the *Wheel*; and the *Weight* (W) is another body, hanging by a string coiled round the *Axle*, and tending to turn the machine round in the opposite direction.



* The other Mechanical Powers are the Lever, the Toothed Wheel, the Wedge, and the Screw.

PROP. X. There is an equilibrium on the Wheel and Axle when the Power is to the Weight as the radius of the Axle is to the radius of the Wheel.

Suppose the strings by which the Power and the Weight hang, to be in the planes of the circles which are those ends of the two cylinders that lie in the same plane*. Since the cylinders have a common axis, their circular ends have a common center; let *O* be it, and join *O* with *M* and *N* the points in which the strings leave (and therefore touch) the circumferences of the circles. *OM* and *ON* are therefore perpendicular to *MP* and *NW*, the lines in which the Power and the Weight act.



Now *P* and *W* act perpendicularly on the arms *OM* and *ON*, and balance each other; and therefore, supposing *MON* to be a lever moveable round *O* as fulcrum,

$P : W :: ON : OM ::$ radius of axle : radius of wheel.

When, therefore, this ratio of the Power to the Weight is fulfilled on the Wheel and Axle, there will be equilibrium. Q. E. D.

[COR. Since *OM* and *ON* are both perpendicular to the vertical lines in which *P* and *W* act, and also pass through the same point *O*, *MON* is a horizontal straight line.]

32. DEF. THE PULLEY consists of *a grooved wheel* moveable round an axis passing through the centre of the wheel, and whose ends are fixed in a frame called *the Block*.

When the Pulley is used as a Mechanical Power, a string, which is fastened at one end, is passed round part of the circumference of the wheel, and the Power is applied at the other end of the string. The Weight is attached to the Block.

PROP. XI. In the single moveable pulley where the strings are parallel, there is an equilibrium when the Power is to the Weight as 1 to 2.

* These circles are *HD* and *AG* in the figure on p. 29.

Let PA, which touches the pulley at A, be the line in which the Power (P) acts. Join A with C the center of the pulley. Produce AC to B. Draw BR touching the circle; it is perpendicular to AB and parallel to AP; this, therefore, will be the other part of the string. Let W, the weight, act at C by means of the block, in the line CW.

The whole being at rest, the string $RBDAP$ must be equally stretched throughout, or motion would ensue. The force upwards, therefore, exerted at the fixed point R is P; for it is the same as that which keeps AP stretched. Art. 14.

Now considering ACB as a straight lever kept at rest round C by forces P and P acting perpendicularly upwards at A and B, the pressure upwards on C will be $2P$ (Prop. II); wherefore the weight (W) which acts downwards at C and is supported by this pressure, is $=2P$;

or $P : W :: 1 : 2$.

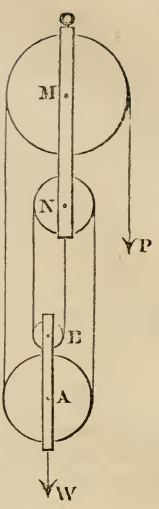
Wherefore, "In the single moveable pulley, &c." Q.E.D.

[Cor. The pressure produced at C by the forces P and P acting at A and B is perpendicular to AB, or parallel to AP and BR (Art. 18. Ax. I.); wherefore the line of action of the Weight is perpendicular to AB, or parallel to AP and BR. And therefore since in a case of equilibrium the strings are parallel to the line in which the weight acts, if the Weight be a heavy body hanging freely from C, AP and BR are vertical.]

33. Prop. XII. In a system in which the same string passes round any number of pulleys, and the parts of it between the pulleys are parallel, there is an equilibrium when Power (P) : Weight (W) :: 1 : number of strings at the lower block.

[In this Proposition, as in Prop. XI., the Power and the Weight are supposed to act in a direction parallel to that of the strings.]

Because there is equilibrium, the portions of the string which lie on either side of any pulley must be pulled by equal forces; and since the outside portion, to which the power is applied, is acted on by a force P, each portion is therefore pulled by a force P.



Wherefore the part of the weight that is sustained at A is (Prop. XI.) $2P$; and the same is true with respect to the pulley B; and so on for every other pulley at the lower block. If, therefore, there be n pulleys at the lower block, W, the whole weight supported, will be $n \times 2P$; or, $W = 2nP$;

$$\therefore \frac{P}{W} = \frac{1}{2n},$$

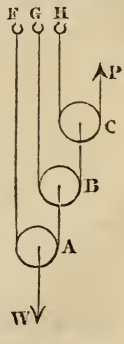
i.e. $P : W :: 1 : 2n :: 1 :$ number of strings at the lower block.

Wherefore, "In a system, &c." Q. E. D.

34. PROP. XIII. In a system where each pulley hangs by a separate string and the strings are parallel, there is an equilibrium when $P : W :: 1 : 2$ raised to that power whose index is the number of moveable pulleys.

[The weight W is supposed to act in a line parallel to the strings.]

In the figure, C being a moveable pulley with parallel strings, there is equilibrium if the pressure downwards at $C = 2P$.



$\therefore$ Tension of $BC = 2P$;

$\therefore$ Weight supported at moveable pulley $B = 2 \times (2P) = 2^2 P =$ tension of AB;

So Weight supported at moveable pulley $A = 2 \times (2^2 P) = 2^3 P$.

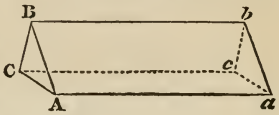
Wherefore, if W, "the Weight," be $= 2^3 P$, there is equilibrium on the system of *three* moveable pulleys here represented.

And in the same manner, if n were the number of moveable pulleys, it would appear by the same mode of reasoning that there would be equilibrium when $W = 2^n P$;

$$\therefore \frac{P}{W} = \frac{1}{2^n}; \qquad \text{or, } P : W :: 1 : 2^n.$$

Wherefore, "In a system where each pulley hangs by a separate string, &c." Q.E.D.

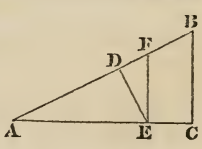
35. DEF. If a prism whose ends are similar and equal right-angled triangles, (C and c being the right angles), be placed so that either of the sides of the prism adjacent to the right angles (as $ACca$) be horizontal, the slant side ($ABba$) is called an *Inclined Plane.*



When the Inclined Plane is used as a Mechanical Power, the Power and the Weight both act in a plane parallel to either end of the prism, and the Inclined Plane is represented by a triangle such as ABC, of which the side AB is called the *Length* of the plane, AC its *Base*, and CB its *Height*. And the plane, by being supposed to be perfectly rigid and inflexible, will be capable of counteracting, and entirely destroying, the effect of any force which acts upon it perpendicularly.

36. PROP. XIV. The weight (W) being on an Inclined Plane and the force (P) acting parallel to the plane, there is an equilibrium when $P : W ::$ the height of the plane : its length.

Let AB be the length of the plane; AC its horizontal base; and BC, perpendicular to AC, its height. Let the Weight (W) act vertically at D, and the Power (P) act in the line DB which is parallel to the plane; Then if $P : W :: BC : AB$, there is equilibrium.



From D draw DE at right angles to AB, meeting the base in E; and from E draw EF vertical, or at right angles to AE.

Then in the triangles *EFD* and *ABC*,

angle *DFE* = angle *ABC*, ∵ *FE*, *BC* are parallel, and *AFB* cuts them;

and angle *FDE* = right angle = angle *BCA*,

therefore angle *DEF* = angle *CAB*, and the triangles are equiangular and similar.

Now, by hypothesis,

$P : W :: BC : AB$

$:: DF : FE$, by equiangular triangles.

But the same effect that is produced on *D* by the two forces which are represented in magnitude and direction by *DF* and *FE*, will be produced by their resultant. But their resultant *DE* (Art. 29. Cor.), being perpendicular to the plane *AB*, will be entirely counteracted and destroyed by that plane, and no motion can be communicated by it to the body at *D*. The body at *D*, therefore, will remain at rest on the inclined plane when acted on by the two forces *P* and *W* which bear to each other the ratio of *BC* to *AB*.

Wherefore, "The Weight, &c." Q.E.D.

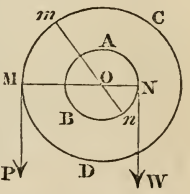
37. DEF. By the VELOCITY of a body in motion is meant the degree of quickness, or speed, with which the body is moving.

[This "degree of quickness" is described, or *measured*, by saying how long the line is that the body moves through in some *given* portion of time. Thus a clear notion would be conveyed of the Velocity of a coach, if it were said to be *nine miles* in *an hour;* the *space* moved over by the coach being *nine miles*, and *an hour* being the *portion of time* during which the motion takes place. If only the space that is described were mentioned, and nothing were said about the time taken up in describing that space, or if the time alone were given, it is evident that no idea at all could be formed of *the degree of quickness*, that is, of the *Velocity*, of the coach's motion.

The Velocity of a body is measured, in mathematical investigations, by the number of *feet* passed over by the body in a *second* of time.

Cor. Since the quicker a body moves the more space it will pass over in a *given* time, it will follow from the observations just made, that *The Velocities of two bodies that move during any* (THE SAME) *time, are as the spaces which the bodies respectively describe in that time.*]

38. Prop. xv. Assuming that the arcs which subtend equal angles at the centers of two circles are as the radii of the circles, to shew that if P and W balance each other on the Wheel and Axle, and the whole be put in motion, $P : W ::$ W's velocity : P's velocity.



Suppose P and W to act at the circumferences of the Wheel and the Axle, in the same plane, perpendicularly to the radii OM and ON (as in Prop. x.); and let the whole be put into motion round the axis so that mOn may become the horizontal diameter. P will now act at m at right angles to Om, and W will act at n at right angles to On; and the velocities of P and W will be as Mm to Nn, since the former of these arcs is the length through which P will have descended in the same time that W has ascended through Nn.

$\therefore$ Vel. of P : vel. of $W :: Mm : Nn$, (Art. 37. Cor.),

$:: OM : ON$ [by the assumption made in the enunciation],

$:: W : P$, by Prop. x; because, by the hypothesis, P and W originally balanced each other.

Wherefore, "Assuming, &c." Q.E.D.

39. Prop. xvi. To shew that if P and W balance each other on the Machines described in Propositions xi, xii, xiii, and xiv, and the whole be put in motion, $P : W ::$ W's velocity in the direc-

tion of gravity : P's velocity [in the direction in which it acts.]

(1) Let C, the center of the moveable pulley (see Fig. Prop. XI.), be raised through any height, as an inch; W will thereby be also raised through an inch, and each of the strings RB and AP will have been shortened an inch; so that if P continue to keep the string tight, it will have moved through two inches in the time that W has been raised one inch.

Now $P : W :: 1 : 2$, Prop. XI,

:: 1 inch : 2 inches,

:: W's velocity : P's velocity; Art. 37. Cor. Q.E.D.

(2) In the system where the same string passes round all the pulleys, and the parts of it between the pulleys are parallel, as in Prop. XII, (see Fig.), if the lower block be raised an inch, each of the strings between the upper and lower pulleys will be shortened an inch, and, there being $2n$ of such strings, P must have moved through $2n$ inches in the time that W moved through one inch, in order to have kept the string tight.

And $P : W :: 1 : 2n$; Prop. XII,

:: 1 inch : $2n$ inches,

:: W's velocity : P's velocity; Art. 37. Cor. Q.E.D.

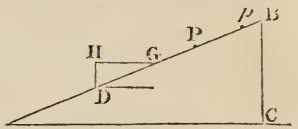
(3) In the system where each pulley hangs by a separate string and the strings are parallel (Prop. XIII), if W be raised through an inch, and P have also moved through such a space that the strings are kept tight, A will have been raised through one inch, and B through two inches (by the first case proved in this Proposition). And B having been raised through two inches, C (by the first case) will have moved through 2×2, or 2^2, inches; and the next moveable pulley (the fourth) would have been raised through 2×2^2, or 2^3, inches. By the same kind of reasoning, if n were the number of *moveable* pulleys, the highest of them would have moved through 2^{n-1} inches, and the end therefore of the string by which P acts would have moved through 2^n inches; but

$P : W :: 1 : 2^n$, Prop. XIII.;

:: 1 inch : 2^n inches

:: W's velocity : P's velocity; Art. 37. Cor. Q. E. D.

(4) Let the weight (W) be kept at rest at D on the inclined plane AB by the power (P) which acts parallel to the plane by means of a string DP. Suppose P, by moving through the space Pp, raises W through an equal space DG. Through G draw GH horizontal, and through D draw DH vertical. Then, by being moved through DG, W has been raised through a vertical height DH; DH being the vertical line which cuts the horizontal lines passing through the two positions of W. Since, therefore, in the time that P moves through a space equal to DG, the vertical height that W moves through is DH, DG is to DH as the velocity of P in the direction of its action is to the velocity of W in the direction of gravity.



Now, in the triangles GDH and ABC,

since GH is parallel to AC, and AG meets them;

$\therefore$ angle DGH = alternate angle BAC;

and since DH, being vertical, is parallel to BC, and DB meets them;

$\therefore$ angle GDH = alternate angle ABC;

$\therefore$ also angle DHG = angle BCA, and the triangles are equiangular, and $\therefore$ similar.

And $P : W :: BC : AB$; Prop. XIV.;

:: $DH : DG$, by similar triangles.

:: W's vel. in direction of gravity : P's vel. in the direction of its action.

Wherefore, "If P and W balance, &c." Q. E. D.

CHAPTER V.

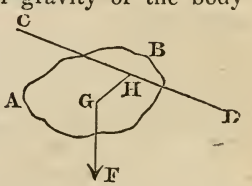
THE CENTER OF GRAVITY.

40. Def. The center of gravity of a body, or of a system of any number of bodies connected together, by inflexible bars which are without weight, is that point upon which the body, or the system, will balance when placed in any position whatever; the point itself being maintained at rest, and the body, or the system, being acted on by no other pressure than that arising from the weight of the matter composing it.

[The Center of Gravity of a body is, as it were, that *fulcrum*, round which the body, when placed in any position, has, of itself, no kind of tendency whatever to turn, although the body be *capable* of being moved in any plane, and in any direction, about that fulcrum.]

41. Prop. xvii. If a body balance itself upon a line in all positions, the center of gravity of the body is in that line*.

If possible, let *G*, the center of gravity of the body *AB*, *not* lie in the line *CD* on which the body balances in all positions; and let the body and line be so placed, that *G* and *CD* may be in the same horizontal plane. From *G* draw *GH* perpendicular to *CD*, and *GF* vertical.



Now since (by Def. Art. 40,) if *G* be supported the body remains at rest when turned round the line into any position, the vertical pressure, therefore, which arises from

* That is to say; If there be a line round which a body can be made to revolve, such that if when the line is put into *any* position, the body, after being made to revolve round the line into *any* position, remains at rest without having any tendency to move, the center of gravity of the body lies in that line.

the weight of the matter composing the body, must always pass through G, or else that pressure could not be rendered inoperative (as it is supposed to be) by G being maintained at rest. Wherefore, in the position of the body represented in the figure, this pressure will act perpendicularly at an arm GH, and have a tendency to turn the body round the point H, which (since H is a point in a line CD fixed in position) may be considered to be a fixed fulcrum. But, by hypothesis, when CD is supported the body balances in *all* positions upon it. Wherefore G does not lie out of the line CD, i. e. it lies in it. Q. E. D.

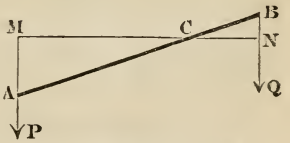
42. PROP. XVIII. To find the center of gravity of two heavy points*, and to shew that the pressure at the center of gravity is equal to the sum of the weights in all positions.

Let P and Q be the weights of two heavy points A and B. Join AB by a straight inflexible bar without weight, and take

$$AC : AB :: Q : P+Q;$$

and $\therefore AC : AB-AC$, or CB,

$$:: Q : (P+Q)-Q, \text{ or } P.$$



Let ACB be in any position. Through C draw MCN horizontal; and through A and B draw the vertical lines MAP and BNQ; these last are the lines in which the weights P and Q act.

Then, the angles at M and N being right angles, and the angle ACM being equal to the opposite angle BCN, the angle CAM is equal to the angle CBN, and the triangles ACM and BCN are equiangular, and $\therefore$ similar.

Now, supposing P to act on the horizontal lever MCN at M at right angles to CM, (Art. 15), and Q to act at N at right angles to CN, since

* The "heavy points" spoken of in this Proposition and the next, are exceedingly small material bodies, and not geometrical points, which (being defined by Euclid to be "without parts") do not possess Length, or Breadth, or Thickness, and consequently can be of no Weight, since they can contain no Matter.

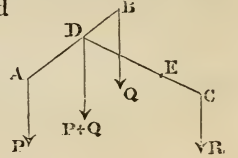
$$P : Q :: CB : CA$$
$$:: CN : CM, \text{ by similar triangles,}$$

therefore P and Q will balance on a fulcrum placed at C, and the pressure on C is $P+Q$, acting vertically. Props. II. and VII.

And this, being true for *any* position, is true for *all* positions of A and B. Wherefore, by the definition of the Center of Gravity, C is the Center of Gravity of A and B; and the pressure at C, in all positions of A and B, is $P+Q$, acting vertically downwards.

43. PROP. XIX. To find the center of gravity of any number of heavy points; and to shew that the pressure on the center of gravity is equal to the sum of the weights in all positions.

Let A, B, C, three heavy points whose weights are P, Q, R, be connected together and placed in any position.



Join AB, and take D a point in AB such that

$$AD : AB :: Q : P+Q;$$

$$\therefore AD : AB-AD, \text{ or } DB, :: Q : (P+Q)-Q, \text{ or } P;$$

therefore D is the center of gravity of A and B, and the pressure produced by P and Q in all positions of the system, is a pressure $P+Q$ acting vertically at D. Prop. XVIII.

Join DC, and take in DC a point E such that

$$DE : DC :: R : P+Q+R;$$
$$\therefore DE : EC :: R : P+Q;$$

therefore E is the center of gravity of the weights $P+Q$ acting at D and R acting at C; and if E be supported, those weights are supported in any position of the system. Since therefore the system will balance itself in all positions on E, that point is its Center of Gravity;—and the Pressure on E is $P+Q+R$.

The construction here applied to a system of three bodies is applicable to a system of any number of bodies.

Wherefore, "The center of gravity of any number of heavy points may always be found, and the pressure on the center of gravity is equal to the sum of the weights." Q. E. D.

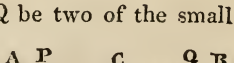
44. [By the definition given in Art. 40, of "the Center of Gravity" of a body, it will be understood that to have a center of gravity a body must have *Weight.* Now in the next two Propositions it is required to find the centers of gravity of a *line,* and of a *plane;* the former of which is defined by Euclid to have *length* merely, without either *breadth* or *thickness;* and the latter, though possessing *length* and *breadth,* is defined to be without *thickness.* A geometrical line, or plane, therefore, *can* have no weight; since there can be no weight where *matter* does not exist, and when *matter* exists under any form whatever, it is of *three* dimensions, or has *length,* and *breadth,* and *thickness.* The line, therefore, and the plane, of which it is required to find the centers of gravity, are not the line and plane of geometry.

But the line of which the center of gravity is determined in the next Proposition, is supposed to be formed of very small equal heavy bodies placed, at the same distance one from the other, along the whole length of the line. And the plane triangle referred to in the next Proposition but one, is supposed to be made up of such lines arranged parallel to any one of the sides of the triangle, and at equal distances one from the other.]

45. Prop. xx. To find the center of gravity of a straight line.

Let AB be a straight line composed of small equal heavy bodies ranged at equal distances one from the other from end to end of it.

Bisect AB in C, and let P and Q be two of the small heavy bodies equally distant from C.



Then P and Q will balance in every position on C. And since the same is true of all such pairs of heavy bodies equidistant from C, the whole line will balance on C in every

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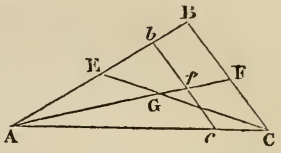
position, and therefore C is the center of gravity of the line.

46. Prop. xxi. To find the center of gravity of a triangle.

Let ABC be a triangle formed of lines ranged, at equal distances, parallel to any one of the sides, the lines themselves being made up of small equal heavy bodies placed at equal distances. Let bfc, parallel to BFC, be one of these lines.



Bisect AB in E and BC in F; join AF, CE by lines intersecting in G;—G is the center of gravity of the triangle.

For $Af : fb :: AF : FB$, (from the equiangular triangles, ABF, Abf),

$$:: AF : FC, \quad \because BF = FC;$$

$:: Af : fc$, (from the equiangular triangles, AFC, Afc).

And since the first and third terms of this proportion are the same, fb is also equal to fc; and therefore the straight line bc would balance in any position on f. Prop. xx.

In the same manner all the other lines parallel to BC might be shewn to balance in any position on the points in which they are cut by AF; therefore the whole triangle would balance on AF in any position, and therefore the center of gravity of the triangle is in AF.

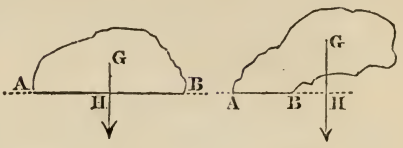
Similarly it may be shewn that the center of gravity of the triangle is in the line CE.

Wherefore G, the intersection of AF and CE, is the center of gravity of the triangle ABC.

47. Prop. xxii. When a body is placed on a horizontal base, it will stand or fall, according as the vertical line drawn from its center of gravity falls within or without the base.

Let GH be a vertical line drawn through G the center

of gravity of the body, and meeting the horizontal plane on which the body is placed in H; and let ABH be any horizontal line drawn through H and terminated at A and B, points in the boundary of the base of the body.



(1) Let H fall *within* the base; it is therefore *between* A and B.

Then AB being considered as a horizontal lever acted on at H (Art. 13) by the weight of the body, no motion can take place round A, because the tendency of the body's weight would be to draw the lever *downwards* round A, a motion which the resistance of the plane will prevent. For the same reason no motion can take place round B; and therefore the body will fall over neither at A nor B.

In the same manner it may be shewn that the body will not fall over at either extremity of *any other* horizontal straight line which is drawn through H, and is terminated by the boundary of the base.

Wherefore the body will remain at rest, when the vertical drawn from its center of gravity falls *within* the base.

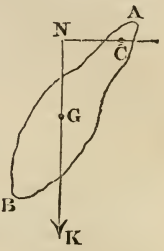
(2) If H fall *without* AB, as in the second figure, no motion of the lever ABH can take place round A as fulcrum, because the effect of the weight acting at H would be to draw AH *downwards*, a motion which the plane prevents. But if B be considered as the fulcrum, the effect of the weight at H would be to make the line AB to move *upwards* from the plane round B, and as there is no force to prevent this motion taking place, the body, though it were prevented from falling over at every other point of the boundary of the base, would still fall over at B. When, therefore, H falls outside the horizontal base, the body cannot stand, but must fall over at some point or other in the boundary of the base.*

Wherefore, "When a body, &c." Q.E.D.

* The demonstration of the second case requires it to be understood that no line whatever can be drawn through H, meeting the circumference of the base in points which are on opposite sides of H.

48. Prop. xxiii. When a body is supended from a point [round which it can swing freely], it will rest with its center of gravity in the vertical line passing through the point of suspension.

Let AB be the body in any position, C the point from which it is suspended, G its center of gravity, NGK a vertical line drawn through G, CN a perpendicular from C on NGK.



Then since the weight of the body acts in the vertical line which passes through G the center of gravity, it may be supposed to be applied at N; and, CN being considered as a lever moveable round C as fulcrum, since there is no force to counteract W acting perpendicularly at N on CN, motion must ensue.

But if NG pass through C, CN vanishes, the weight is sustained by the immoveable fulcrum C, and the body is at rest.

Wherefore, "When a body, &c." Q.E.D. *End.*

Definition of Fluid.

A Fluid is a body composed of materia particles, so constituted, that if any prefsure be communicated to the fluid this prefsure is transmitted by the flu undiminished & unimpaired in every direct

THE ELEMENTS

OF

HYDROSTATICS.

The explanatory articles and paragraphs which are enclosed in brackets form no part of the Course adopted by the University. Those in smaller type are illustrative of the Definitions and Propositions they immediately follow. The whole of the extra matter so introduced forms but a very small, and a very easy, portion of the book; and the Student who comes to this subject for the first time had better not omit it.

CHAPTER I.

1. The Science of Hydrostatics investigates the conditions fulfilled by the pressures produced by fluids when in a state of rest.

2. Def. A Fluid is a material body of which the parts can be divided in any direction, and moved among one another by any force, however small.

[Mercury, Water, Air, Gas, Steam, are all instances of fluids.]

3. Fluids have been divided into *Elastic*, and *Non-elastic*.

(1) Elastic Fluids are those of which the dimensions are increased or diminished when the pressure upon them is diminished or increased.

[Air is an elastic fluid, as will hereafter be proved. So also is Gas, and Steam.]

(2) Non-Elastic Fluids are those of which the dimensions are not diminished by the addition of pressure, or increased by the withdrawal of pressure.

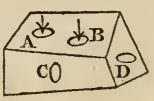
[Water, Mercury, and probably all other liquids, are compressible, but in a slight degree only. The resistance, however, which they offer to compression is so great, that the conclusions obtained on the supposition of their being entirely incompressible are free from any sensible error, except in cases where the pressure is exceedingly great.]

CHAPTER II.

PRESSURE OF NON-ELASTIC FLUIDS.

4. PROP. I. FLUIDS press equally in all directions.

Let a close vessel of any shape be filled with fluid, and let *A*, *B*, *C* and *D*, *similar* and *equal* portions (of any size and shape) of the sides of the vessel, be removed, and their places supplied by plugs fitting the orifices exactly and acted upon by pressures just sufficient to keep them at rest. Then if an additional force *P* be applied to any one of the plugs, it is found, by experiment, that an *equal* additional pressure must be applied to *every other* of the plugs, to prevent the fluid from bursting out.

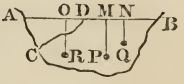


This proves that a pressure communicated to the surface of a fluid at rest is transmitted by the fluid, undiminished and unimpaired, to every other equal and similar portion of the surface by which the fluid is bounded.

Also, whatever be the *directions* in which any of these plugs are inserted through the sides of the vessel, the same quantity of pressure is required to keep each of them at rest; which shews that, at the same point in a fluid, the fluid presses with *equal* force in *all* directions.

Wherefore, "Fluids press equally in all directions." Q. E. D.

5. Prop. II. The pressure upon any particle of a fluid of uniform density is proportional to its depth below the surface of the fluid.



Let the vertical lines PM and QN be drawn through P and Q two particles situated in the interior of an uniformly dense fluid which is at rest, and let them meet the surface of the fluid in the points M and N.

Suppose the pressures on P and Q to be produced by the weight of fluid particles, of the same magnitude as P and Q, placed touching each other along the lines PM and QN. Then, since the fluid is of uniform density, the weights of such lines of particles will be proportional to their lengths.

$\therefore$ Pressure on the particle P

: pressure on the particle Q

:: weight of the line PM of particles

: weight of the line QN of particles

:: $PM : QN$.

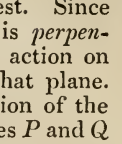
Again; the fluid being at rest, the pressure of any part of it will not be affected if the particles (which are at rest) of the fluid upon which the pressure acts, be supposed to become connected with one another. Whence it will follow, that whether there be an uninterrupted line of fluid particles (as RO) reaching from a particle R to the surface of the fluid, or part of the line be cut off by a portion ACD of the fluid becoming solid, the pressure upon the particle R will still be the same.

Wherefore, "The pressure, &c." Q. E. D.

6. Prop. III. When a fluid is at rest its surface is horizontal.

[The manner in which the matter composing non-elastic fluids acts is twofold; first, by the particles pressing with their weights on those immediately below them; and, second, by the property these particles possess of communicating, in all directions and without diminution, any pressure to which they are subjected.]

Imagine two contiguous particles P and Q, situated in the same horizontal plane in a fluid at rest. Since the action of gravity on these particles is *perpendicular* to the plane in which they lie, its action on either of them can produce no effect *in* that plane. It is in consequence, therefore, of the action of the fluid which surrounds them, that the particles P and Q have no horizontal motion: and this action must be the same on each, or motion would ensue.



But since fluids press equally in *all* directions (Prop. I.), and the *horizontal* pressures on P and Q cannot but be equal, the vertical pressures on them, which are as the distances Pa and Qb below the surface of the fluid (Prop. II.), must also be equal; and therefore

$$Pa = Qb;$$

$\therefore$ ab is parallel to PQ, and is $\therefore$ horizontal.

And since, in like manner, the line joining *any two* adjacent particles in the surface of the fluid may be shewn to be horizontal, the surface itself is horizontal.

Wherefore, "When a fluid, &c." Q. E. D.

7. PROP. IV. If a vessel, the bottom of which is horizontal and the sides vertical, be filled with fluid, the pressure on the bottom will be equal to the weight of the fluid.

The sides of the vessel being vertical, and the whole of the fluid being conceived to be made up of vertical straight lines of fluid particles, each of these lines will press vertically with its weight, and the sum of these vertical pressures will be the weight of the fluid in the vessel. Now the base of the vessel, being horizontal, will counterbalance all the vertical pressures upon it, and destroy them entirely. The pressure, therefore, sustained by the horizontal base of a vessel whose sides are vertical, is equal to the weight of the fluid in the vessel. Q. E. D.

8. [From Propositions II and IV the following conclusion may be drawn.

The pressure exercised by a fluid on any horizontal plane placed in it, is equal to the weight of a column of the fluid whose base is the area of the plane, and whose height is the depth of the plane below the horizontal surface of the fluid.

Let AB and CD be two equal areas in the same horizontal plane. Since the pressure is the same on every point in the same horizontal plane, and (the areas AB and CD being equal) the number of particles in the plane CD is the same as that in the plane AB, therefore the pressure on AB = pressure on CD.

Let AB be an area such that a vertical line of fluid particles reaches from every particle in it to the surface, and suppose the rest of the fluid to become solid; $EABF$ then becomes a vessel with vertical sides and horizontal base, and the pressure on AB = weight of the column EB of fluid. Prop. IV.

Therefore, the pressure on any *horizontal* area CD = weight of a column of the fluid whose base is CD, and whose height is the depth of CD below the horizontal plane of the surface of the fluid in the vessel. Q. E. D.]

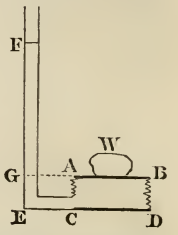
9. [From Art. 43 of the MECHANICS it appears that the pressure on the Center of Gravity of a system of bodies considered as heavy points, is the same as if the weights of the bodies were collected at it. So long therefore as the Quantity of Matter in the system remains the same, the amount of Pressure produced by the weights of the several parts of the system is the same whatever be the manner in which they may be arranged.

Now though at first sight it might appear probable, that the pressure on the bottom or the sides of a vessel filled with fluid would depend, somehow or other, on the *quantity* of the fluid by which that pressure is produced, such is by no means the case; for it is found, both from experience and by theory, that the pressure produced by a fluid, at rest, on the surface of the vessel containing it, is not dependent on the quantity of the fluid; a fact apparently so much at variance with the law governing the amount of pressure produced by Solid bodies, that it has been called THE HYDROSTATIC PARADOX.]

10. Prop. v. To explain the Hydrostatic Paradox.

The Hydrostatic Paradox is this; that "any Pressure, however small, may, by means of a fluid, be made to counterbalance *any other* Pressure, however large; and whether the quantity of fluid employed be great or small, it is possible to produce this result."

The top and bottom of a vessel AD are boards which are connected together by leathern sides. The vessel communicates with a vertical tube EF of small uniform bore, by means of a horizontal pipe CE. Let (A) be the number of square inches in the area of the board AB, and (a) the number in the area of a horizontal section of the pipe EF.



Let AB be held in a horizontal position, and water be poured into the tube until it just rises in AD to AB. The water will therefore rise in the tube to G, a point in the same horizontal plane as AB; Prop. III.

If now there be a heavy weight W laid upon AB, it is found that it can be supported at rest by a small quantity FG of water poured into the tube. To shew the reason of this.

Since the sides of the tube FG are vertical, the pressure on the horizontal section of the tube at G is the weight of the fluid column FG. Suppose w to be this weight,

Then, since the pressures at every point in the same horizontal plane of a fluid at rest are the same,

$\therefore$ Pressure on the area (A) of AB : pressure on the area (a) at $G :: A : a$,

$$\text{i. e., } W : w :: A : a;$$

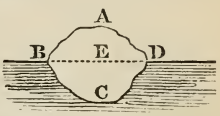
$$\therefore\ w = W \times \frac{a}{A}.$$

If therefore W and A be given, w may be made as small as may be wished by diminishing a.

Whether the pressure on the area (a) at G be produced by the weight of the fluid column GF, or by means of a plug acted on by some force, the pressures on the areas (a) at G and (A) at AB will still bear to one another the ratio of a to A; a ratio which is wholly independent of the quantity of fluid contained in the vessel.

11. Prop. vi. If a body float on a fluid, it displaces as much of the fluid as is equal in weight to the weight of the body; and it presses downwards, and is pressed upwards, with a force equal to the weight of the fluid displaced.

Let $ABCD$ be a body that displaces the portion $BEDCB$ of the fluid on which it floats. The weight of the floating body produces a pressure which acts vertically downwards. Therefore, the effective pressure of the fluid, by which the body is kept at rest, must act vertically upwards, and be equal to the weight it balances.



Now let the floating body be removed, and the space $BEDCB$ be filled with fluid the same as that on which the body floated. The equilibrium of the fluid will not be disturbed, neither will the pressure of that part of it which was formerly in contact with the surface of the floating body be altered, if the parts of the fluid now in $BEDCB$ be supposed to become permanently connected with one another, and so to form a solid.

Let this take place. The pressure downwards of $BEDCB$ is its weight; and since this pressure is counteracted by the same sustaining power as that which balanced the weight of the floating body, it follows that the weight of the floating body is equal to the weight of the fluid it displaces.

Wherefore, "If a body float on a fluid, &c." Q. E. D.

CHAPTER III.

SPECIFIC GRAVITIES.

12. THE BULK, or VOLUME, or CONTENT, or MAGNITUDE of a solid body is measured by the number of times the body is greater than some particular body previously fixed upon to compare the sizes of other bodies by, which is called "the unit of solid measurement."

A cube of which an edge is an inch in length is called "a cubic inch." In the following pages *a cubic inch* will be taken for the unit of measurement; so that when it is said that M is the *bulk, volume, content,* or *magnitude,* of a body, it is meant that the body is M cubic inches in size.

13. DEF. The SPECIFIC GRAVITY of any substance is the weight of an unit of its magnitude.

If, (as stated in the last Article,) the magnitude of a body be measured by the number of *cubic inches* it contains, and the weight of a cubic inch of the substance the body is composed of be given in *grains,* then it will follow, that, S being taken to represent the Specific Gravity of any substance, S is the *number* of *grains* that *one cubic inch* of that substance *weighs.**

* The Tables commonly called "TABLES OF SPECIFIC GRAVITIES" give the ratios which the weights of bulks of various substances bear to *equal* bulks of Water.

It having been found, by experiment, that the weight of a piece of Iron : the weight of a bulk of Water *of the same size* :: 7·8 : 1, and that the weight of a piece of Silver : weight of an equal bulk of Water :: 10·5 : 1, and so for other substances,—tables have been formed in which the numbers 1, 7·8, 10·5, &c., are placed opposite the words "Water" "Iron" "Silver" &c. By means of these tables, (as will be shewn), the weight of any bulk of any of the substances so registered can be determined, if only the weight be known of some particular magnitude (such as a cubic inch) of some one of them.

[The numbers given in these Tables are called, by some writers, the "Specific Gravities" of the several substances registered; but the enunciation of Prop. VII. Art. 14, prevents their being so called here.]

14. Prop. vii. If M be the Magnitude of a body, S its Specific Gravity, and W its Weight, $W = MS$.

Let M = number of cubic inches in the body.

S = Specific Gravity of the body; i. e. let S be the number of grains that one cubic inch of it weighs.

W = number of grains that the whole body weighs.

Then since $\underset{\text{grains}}{W} : \underset{\text{grains}}{S} :: \underset{\text{c. inches}}{M} : \underset{\text{c. inch}}{1}$;

$$\therefore W = M \times S. \qquad \text{Q. E. D.}$$

"The Tables of Specific Gravities" give

Platina	21·5	Zinc	6·9
Gold	19·4	Sea Water	1·027
Mercury	13·6	Water	1·
Lead	11·4	Proof Spirit	0·930
Silver	10·5	Pure Alcohol	0·825
Copper	8·9	Air, at the surface of the Earth,—the average	0·00125
Iron	7·8		
Tin	7·3		

Hence it may be shewn that "the weights of any two substances of equal bulk are in the ratio of the numbers given by the Tables as corresponding to the substances."

For w, w', w'' being the respective weights of equal bulks of Water, Iron, and Silver, since, as explained above,

$$w' : w :: 7{\cdot}8 : 1, \text{ and } w : w'' :: 1 : 10{\cdot}5;$$

therefore, compounding these proportions,

$$w' : w'' :: 7{\cdot}8 : 10{\cdot}5.$$

So that if it be required to find the weight of a cubic foot of Iron, having given that the weight of 10 cubic inches of Silver is 61 ounces nearly,

$\therefore$ Weight of a cubic foot (or $12 \times 12 \times 12$ cubic inches) of Silver,

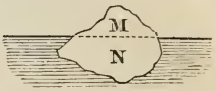
$$= 12 \times 12 \times 12 \times \frac{61}{10} \text{ ounces};$$

$\therefore$ Weight of a cubic foot of Iron $= \dfrac{12 \times 12 \times 12 \times 61}{10} \times \dfrac{7{\cdot}8}{10{\cdot}5}$ oz.

$$= \frac{12 \times 12 \times 12 \times 61 \times 7{\cdot}8}{105} = 7830 \text{ ounces, nearly.}$$

15. Prop. viii. When a body of uniform density floats on a fluid, the part immersed : the whole body :: the specific gravity of the body : the specific gravity of the fluid.

Let a solid of uniform density float on a fluid with M cubic inches of it above the horizontal plane of the surface, and N cubic inches below that surface.



Let S = S. G. (Specific Gravity) of the solid; S' = S. G. of the fluid.

Then, $(M+N) \times S$ = weight of the solid. Prop. vii.

$N \times S' =$ ..... fluid displaced.

But, because the body floats, these two weights are equal, Prop. vi.;

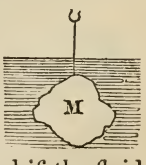
$$\therefore N \,.\, S' = (M+N) \,.\, S; \text{ and } \therefore \frac{N}{M+N} = \frac{S}{S'},$$

or $N : M+N :: S : S';$

i. e.; The part of it immersed : the whole of the body :: S. G. of the body : S. G. of the fluid. Q. E. D.

16. Prop. ix. When a body is *immersed* in a fluid, the weight lost : whole weight of the body :: the specific gravity of the fluid : the specific gravity of the body.

Let M be the number of cubic inches contained in a body of uniform density which is wholly immersed in a fluid; S the S. G. of the body, S' the S. G of the fluid.



Then the pressure downwards of the solid is its weight; and if the solid be removed, and the space it filled be occupied by an equal bulk of the fluid, equilibrium will still remain. And if the fluid so added become solid, the equilibrium will continue, and the pressure upwards of the surrounding fluid will remain the same as before.

Now the pressure downwards produced by the fluid that becomes solid is its weight. And since the pressure upwards of the surrounding fluid supports this weight, that pressure must be exactly equal and opposite to it.

The pressure downwards, therefore, before it was immersed, of the original solid, (i. e. its weight MS, Prop. VII.), must, by the solid having been immersed, have been diminished by a pressure upwards, arising from the surrounding fluid, exactly equal to the weight of the fluid displaced; which weight, by Prop. VII. is equal to MS'.

$\therefore$ Weight lost by the body : the whole weight of the body :: MS' : MS

:: S' : S

:: S.G. of fluid : S.G. of body. Q.E.D.

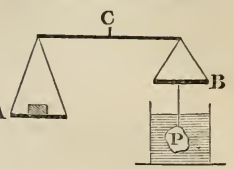
17. [It appears from the proof of the last Proposition, that the pressure of a fluid on a body wholly immersed in it acts vertically upwards, and is equal to the weight of the fluid which the body displaces.

If this pressure be less than the weight of the body,—(that is, if the S.G. of the fluid be less than that of the solid)—the pressure downwards arising from the weight of the solid will be greater than the pressure of the surrounding fluid upwards, and the body will therefore sink to the bottom of the vessel.

But if the pressure of the fluid upwards be greater than the weight of the body immersed,—(that is, if the S.G. of the fluid be greater than that of the solid)—the pressure upwards will be greater than the pressure downwards, and the body will therefore rise until the conditions of Proposition VIII are fulfilled, and will then float.]

18. PROP. X. (1) To describe the Hydrostatic Balance; and (2) to shew how the Specific Gravity of a body may be found by it,—*first*, when its Specific Gravity is *greater* than that of the fluid in which it is weighed,—and *second*, when it is *less*.

(1) The Hydrostatic Balance is the common balance with a hook attached to the under part of one of its scales, so that bodies may be weighed either by putting them into the scale, or by suspending them from it and letting them be immersed in a fluid, as here represented.



(2) First, let the S. G. (Specific Gravity) of the body be *greater* than that of the fluid.

Since the S.G. of the solid is greater than that of the fluid, the body will sink in the fluid. Prop. IX. Cor.

Let S = S. G. of the solid, S' = S. G. of the fluid.

W = weight required to balance the body when placed *in* the scale.

W' = weight required when the body is immersed.

$\therefore W - W'$ = diminution of the body's weight in consequence of immersion.

Therefore, by Prop. IX.,

$$W : W - W' :: S : S';$$

$$\therefore S = \frac{WS'}{W - W'}.$$

Whence, W and W' being known weights, S may be determined if S' be given.

Next, let the S.G. of the body be *less* than the S.G. of the fluid.

When the body, in this case, is forced under the surface of the fluid, the pressure downwards on it (which is the weight W of the body) being less than the pressure upwards (which is the weight of a quantity of fluid equal in magnitude to the fluid displaced) the body must on the whole be acted on by a pressure upwards, which is equal to the weight of the fluid displaced *minus* the weight of the solid.

Let there be taken a body Q, of greater S.G. than that of the fluid, and large enough to sink both itself and the body P whose S.G. is required when P is attached to

W of the body; i.e. the weight of the whole body when placed in the scale, which is denoted by W

it. Let Q be first immersed by itself in the fluid, and balanced by weights in A. Next, leaving the weights in A, let P be attached to Q and both of them be immersed. The scale A will now preponderate, and to restore the equilibrium a portion (W') of the weights in it will have to be removed. Wherefore, (W being the weight of the body P when placed *in* the scale B),

W' = whole *effective* pressure on P when immersed; which acts upwards;

= weight of an equal bulk of fluid *minus* the weight of P (W);

$\therefore$ weight of a bulk of fluid equal to that of $P = W + W'$.

Now since by Prop. VII. $W = M \times S$, therefore when the magnitudes of the bodies are the same, $W \propto S$; *i.e.* W' varies as S.

$\therefore$ weight of P : weight of an equal bulk of fluid

:: S. G. of solid : S. G. of fluid

or, $W : W + W' :: S : S'$;

$$\therefore S = \frac{WS'}{W + W'}.$$

Whence, (W and W' being known weights), S can be found if S' be given.

19. Prop. XI. (1) To describe the common Hydrometer, and (2) to shew how to compare the Specific Gravities of two fluids by it.

(1) The common Hydrometer consists of two spheres attached to each other, and a long uniform slender stem, which, if produced, would pass through the centers of both of them. The upper sphere is hollow; and the lower, being filled with small shot or mercury, serves as ballast for the instrument, and makes it float steadily in a vertical position when put into a fluid. The stem is graduated by divisions of equal lengths.

(2) The Hydrometer is made lighter than a bulk equal to that of the instrument of any of the fluids whose Specific Gravities it is used to compare.

Suppose the bulk of the portion of the stem included between every two graduations to be one four thousandth part of the bulk of the whole

instrument. When the Hydrometer is put into a fluid whose S. G. is S, let 20 divisions be above the surface; and when it is put into a fluid whose S. G. is S', let there be 30 divisions out.

Now the *weights* displaced of the fluids are in both cases the same, namely the weight of the instrument, (Prop. VI.). If M and M', therefore, be the magnitudes of the fluids displaced, by Prop. VII.,

$$M \times S = \text{weight of the Hydrometer} = M' \times S';$$

$$\therefore S : S' :: M' : M$$

$$:: 4000-30 : 4000-20$$

$$:: 3970 : 3980;$$

and the ratio of the Specific Gravities of the two fluids is thus determined.

[A mark P is made at the point in the stem to which the instrument sinks in a fluid called "Proof Spirit", which is a mixture consisting of equal *weights*,—(*not* equal *magnitudes*),—of pure Alcohol and of Water. Alcohol being lighter than Water, if a mixture of these two fluids contain a greater weight of the former than it does of the latter, it will be lighter than Proof Spirit, and the Hydrometer therefore will displace a greater bulk of it than it does of Proof Spirit, that is, it will sink deeper in the mixture than in Proof Spirit. Wherefore the surface of such a mixture will rise to a higher point in the stem than P, and in such case the mixture is said to be "*above proof*."

On the other hand, if the weight of the Water contained in the mixture be greater than that of the pure Alcohol, the Hydrometer will not sink so low as to the point P, and the fluid is then said to be "*below proof*."

When equal weights of pure Alcohol and of Water are mixed, the ratio which the bulk of the Alcohol bears to that of the Water does not differ much from 3 : 2. Out of 100 cubic inches of Proof Spirit, 56.36 are pure Alcohol, and 43.64 are Water.]

CHAPTER IV.

ELASTIC FLUIDS.

20. Prop. xii. Air has Weight.

This is proved by these experiments.

If a close vessel be weighed after the air has been exhausted from it, and again when the air has been admitted, it is found that the weight is greater in the latter than in the former case.

If a bladder be weighed in a vessel from which all the air has been exhausted [by a method which will be described hereafter], first when the bladder contains no air, and again after air has been forced into it, the weight required to balance the bladder is greater in the latter than in the former case.

Whence it is concluded that 'Air has weight.'

21. Prop. xiii. The elastic force of Air at a given temperature varies as the density.

This is proved by experiment.

Let $ABCD$ be a tube, of which the bore is uniform (its perpendicular section being an area of a square inches), and the legs AB and CD are vertical. The end A of the tube is open, and D is closed.

A quantity of air is retained in the shorter leg by some mercury which rises to E and F in the two legs respectively. Through E is drawn the horizontal line Ee.

Then, The weight of the mercurial column Fe

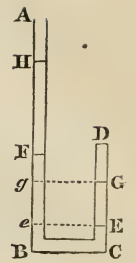
= pressure *downwards* at e on a surface a (by Prop. iv.), the portion of the tube being vertical,

= ..... *upwards* at e.....; Prop. i.

= ..... *upwards* at E.....; Prop. ii.

= pressure by the air in DE on the surface a of mercury with which it is in contact;

[because, the whole being at rest, the pressures upwards and downwards on the same horizontal plane must be equal.]



Similarly, when more mercury is poured in, if its surfaces stand at G and H in the two legs, it follows, drawing Gg horizontally through G,

Weight of the mercurial column Hg = pressure by the air in DG on a portion a of the surface of mercury with which it is in contact.

Now if the lengths ED, Fe, GD, Hg be measured, it is invariably found, however the quantities of air and of mercury used in the experiment be altered, that

$$DE : DG :: Hg : Fe;$$

$\therefore$ content of DE : content of DG :: content of Hg : content of Fe.

But since the same quantity of air is contained in DG that there was in DE,

$\therefore$ Density of the air in DE : density of the air in DG

:: content of DG : content of DE

:: content of Fe : content of Hg

:: weight of mercury in Fe : weight of mercury in Hg,

:: pressure of air in DE on a portion a of surface : pressure of air in DG on an equal portion of surface;—[as shewn above.]

Whence it appears that

The pressure which air exercises on a given surface varies as the density of the air. Q. E. D.

[Note. In the experiment here described the longer leg of the tube is placed in a vessel from which the air has been exhausted, so that there is no pressure at all on the surface of the mercury in the longer leg.

If A communicated with the open air, the surrounding atmosphere would press on the surface of the mercury in AB and would have to be taken into consideration in estimating the whole amount of pressure sustained by an area a placed at e or g.]

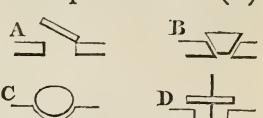
22. PROP. XIV. The elastic force of the air is increased by an increase of temperature.

This is proved by experiment.

If a bladder be partially filled with air and then brought near the fire, the included air soon expands, and the bladder becomes fully distended. On the included air cooling down, the bladder returns to its original flaccid state.

23. DEFS. (1) A VALVE is a kind of door which fits an orifice, such that being pressed by a fluid on one side, it opens and allows the fluid to pass through, but which keeps the orifice tightly closed if the fluid press on the other side.

[Valves are of various forms,—a flap of leather (*A*) fastened at one edge,—a frustrum of a cone (*B*) made of metal,—a sphere (*C*),—or a plate of metal (*D*) with an axis passing perpendicularly through it. By any of these contrivances the flow of a fluid upwards would be prevented only by the weight of the valve; but a rush of fluid from above would carry the valve along with it, and keep the orifice which the valve fits completely closed.

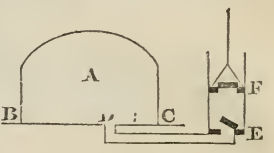


When the fluids employed are very rare,—like air or gas,—the valves are generally made of flaps of oiled or varnished silk, which being attached at two or three points to the surfaces in which the orifices are situated, are raised by a very slight pressure, and so allow fluids of exceedingly small densities to pass under them.]

(2) A PISTON is a plug, mostly cylindrical, which, by means of a rod attached to it, can be made to play within a tube which it fits accurately.

24. Prop. xv. To describe the construction of the common Air Pump and its operation.

Description. The Air Pump consists of a glass vessel *A*, called the *Receiver*, made to fit a table *BC* so as to be air tight. A tube *DE* connects the Receiver with a cylinder *EF*, called the *Barrel*. At the bottom of the Barrel there is a valve *E* opening upwards, and a piston *F* (also furnished with a valve opening upwards) plays within the Barrel.



Operation. Let the piston *F* be at its highest point, and the instrument filled with air the same as that of the surrounding atmosphere.

When the piston is driven down, the air at first in the barrel is condensed and its pressure therefore increased. The valve *E* is kept closed, and the valve in *F* being pressed on the under surface more strongly than on the upper, opens and allows the air in the barrel to escape through it.

Next, on raising the piston, the external air keeps *F* closed, and, there now being no air in *EF*, the pressure on the under surface of the valve at *E* will open that valve, and allow air from the receiver and pipe to flow into the barrel. The figure represents the instrument during the *ascent* of the piston.

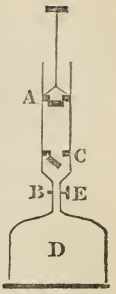
When the piston descends again, the air in *EF* being condensed closes the valve at *E*, and at length opens the valve at *F*, and so escapes; and on the piston reascending, more air rushes from the receiver into the barrel.

The process may be repeated until the air in the receiver becomes so rare that its pressure is insufficient to overcome the weight of the valve at *E*.

25. Prop. xvi. To describe the construction of the Condenser and its operation.

Construction. The Condenser is a *Barrel AB* furnished with a piston *A* that has a valve in it opening downwards; at the bottom of the Barrel there is a

like valve *C*. The neck of the Barrel communicates with an air-tight vessel *D*, called the *Receiver*.



Operation. Let the instrument be filled with common air, and the piston be at its greatest height. On the piston being forced down, the air in the barrel is condensed, and its pressure being therefore increased, (Prop. xiii.) it keeps the valve *A* closed, opens the valve *C*, and escapes into the Receiver. The figure represents the instrument during the *descent* of the piston.

On the piston's ascending, the air in the receiver keeps *C* closed, and there now being no pressure on the under surface of the valve *A*, the external air opens the valve, and the barrel becomes filled with air, which may be driven into the receiver by forcing down the piston again. And in this manner the condensation of the air in the receiver may be carried on to any extent required.

[The communication between the barrel and the receiver can be cut off at pleasure by means of a stopcock at *E*; and the barrel is made to screw off and on at its lower end.]

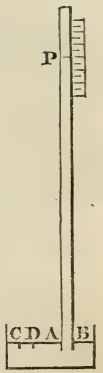
26. Prop. xvii. To explain the construction of the common Barometer, and to shew that the mercury is sustained in it by the pressure of the air on the surface of the mercury in the basin.

The Barometer is an instrument for measuring the pressure of the air. It consists of a glass tube (see fig. Prop. xviii.) of uniform bore, closed at one end, and not less than 33 or 34 inches long. This tube is filled with mercury, and the open end, being first stopped with the finger, is placed below the surface of some mercury in a basin, when the tube being held in a vertical position and the finger being withdrawn, the mercury in the tube subsides, and stands at a height above the mercury in the basin which varies on different days from about 28 to 32 inches. A scale of a few inches is attached to the upper part of the tube, to shew the height at which the mercury may be standing at any time.

That the column of mercury is supported in the tube by the pressure of the atmosphere appears from this experiment. When the whole is put into the receiver of an air pump, the mercury in the tube sinks more and more for every barrel of air that is pumped out, until at length it is all but on a level with the surface of the mercury in the basin. But on readmitting the air into the receiver the mercury in the tube rises to its former level.

27. PROP. XVIII. The pressure of the atmosphere is accurately measured by the weight of the column of mercury in the [vertical tube of the] Barometer.

Let P be the surface of the mercury in the vertical tube; AB the section of the tube made by the horizontal surface of the mercury in the basin; CD a portion of area in that surface equal to AB.



Then, Weight of mercury contained in PB
= pressure downwards on the area AB, Prop. IV;
= pressure upwards on AB,
= pressure upwards on CD; since AB and CD are equal areas situated in the same horizontal plane of a fluid at rest, Prop. II.

But the fluid being at rest, the pressure *downwards* on CD, (which arises from the pressure of the air in contact with it), is equal to the pressure *upwards* on it.

Wherefore the pressure of the atmosphere on any area CD is equal to the weight of the column of mercury supported in the [vertical] tube of the barometer, of a base AB which is equal to that area.

Wherefore, "The Pressure, &c." Q. E. D.

[COR. 1. By Art. 8. the pressure of a fluid on a horizontal plane immersed in it was shewn to be the weight of a column of the fluid whose base is equal to the area of the plane and whose height is the depth of the plane below the surface of the fluid. Wherefore, the pressure exerted by the atmosphere on such an area,—being measured by the

weight of the vertical column of mercury (of equal section) that is supported in the barometer,—is equal to the weight of a column of mercury whose base is equal to the plane acted upon, and whose height is the same as that of the column of mercury supported in the vertical tube of the barometer.]

[Cor. 2. The pressure of the air being measured by the weight of the column of fluid which it supports, the barometer might be filled with any fluid whatever. But mercury being by far the heaviest fluid known, the column of mercury required to produce a given pressure is very much shorter than if any other fluid were employed.

For example, if it took 30 inches of mercury to balance the pressure of the air, then since mercury is 13·6 times heavier than water (see Note. Art. 13.), it would take a column of water $13{\cdot}6 \times 30$ inches, or 34 feet high, to produce the same effect.]

[Cor. 3. The mercury in the barometer standing at 30 inches, the pressure of the air on a surface of the area of a square inch is equal to the weight of a column of mercury 30 inches long and whose base is a square inch; i. e. to the weight of 30×1, or 30, cubic inches of Mercury.—*To find how much this pressure amounts to.*

The weight of a cubic foot of Water is found to be 1000 ounces avoirdupois;

$\therefore$ the weight of a cubic foot of Mercury $= 1000 \times 13{\cdot}6$ oz. (Note to Art. 13.)

$\therefore$ 1, or $12 \times 12 \times 12$: 30 :: $1000 \times 13{\cdot}6$: weight of 30 cubic inches of mercury, which $\therefore = \dfrac{13600 \times 30}{12 \times 12 \times 12}$ ounces = nearly 236 oz. or $14\frac{3}{4}$ lbs.]

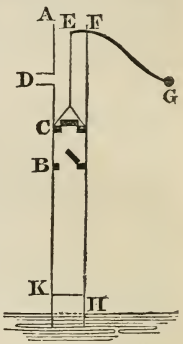
[Cor. 4. In Chapters I and II the pressure of the air on the surface of the fluid contained in an open vessel has been left out of consideration; in other words, the experiments there described were supposed to be made in the exhausted receiver of an air-pump.

From Cor. 2 it appears, that when Water is the fluid employed, the pressure of the air on its surface is equivalent, (when the mercurial barometer stands at 30 inches), to that which would be produced by a head of water 34

feet deep. In estimating, therefore, the pressure on any surface placed at a given depth below the surface of water, this large additional pressure, (amounting by Cor. 3 to more than 14 lbs. on every square inch of surface subjected to it,) must not be left out of the account.]

28. Prop. xix. To describe the construction of the Common Pump and its Operation.

Construction. In the *Common Pump* two hollow cylinders, AB and BH, having the same axis, are connected together, and at their junction a valve B opening upwards is placed. The upper (AB) of these cylinders is called "*the Body of the Pump*"; and in it a piston C, containing a valve opening upwards, plays by means of a rod attached to the end E of a lever EFG whose fulcrum is F. A spout D is placed just above the highest point to which this piston ascends. The lower cylinder (CH), which is called "the *Suction Pipe*," reaches below the surface H of a well of water.



Operation. Let the piston be at B and the suction tube full of air. As C is raised, the pressure of the external air keeps the valve in C closed, and a vacuum between B and C being consequently made, the air in HB, pressing against the under surface of the valve at B, opens it, and escapes into BC.

The air, therefore, which before the ascent of the piston occupied the space BH, now occupies the greater space CBH, and so becoming less dense than before, exercises a less pressure on the surface of the water at H; Prop. xiii. Wherefore, since the external air continues to exercise the same pressure as before on the surface of the water in the well, it will force up water into the suction pipe to a height K such that the pressure of the air in CK, together with the pressure arising from the column of water KH, produces the same effect on the section of the water in the suction pipe at H, as the external air does on an equal area situated in the surface of the water in the well.

When C has come to the highest point of its ascent, and equilibrium exists between the pressure of the external air on

the surface of the water in the well on the one hand, and the pressure of the air in CK together with that arising from the fluid column KH on the other, the valve B, being equally pressed on its upper and its under surfaces, will fall down by its own weight. The piston C is then forced down; the air in CB is condensed until its pressure becomes greater than that of the external air, when it opens the valve in C and escapes.

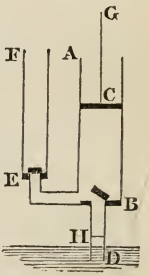
When C is raised again, the same circumstances recur. The water rises a little higher in the suction pipe at every stroke of the piston, and at last flows through B; and on C descending again, it gets above the valve C, and is brought up to the spout at D.

[As the average pressure of the atmosphere will not support a column of water more than 34 feet high, if the valve B be more than 34 feet above the surface of the water in the well, the pressure of the external air in that case not being sufficient to force the water up so high as B, the pump will not work.

N. B. The figure represents the pump during an *ascent* of the piston, when (air, or water, flowing through it) the valve at B is *open*, and that at C is *shut*.]

29. Prop. xx. To describe the construction of the Forcing Pump, and its operation.

Construction. The Forcing Pump consists of a *Barrel* AB in which a solid piston C works by means of a rod GC; BD is a *Suction pipe* reaching below the surface D of a well of water; BE a pipe connecting BC with a vessel EF; at B and E valves are placed, opening upwards.



Operation. Suppose the piston C to be at its greatest height, and the pump full of air. When C descends, the air in AB is condensed, and its pressure being increased (Prop. xiii.), the valve E is opened, and the air that was at first in AB is forced into EF.

On the piston reascending, the air in EBC being now rendered less dense than that in EF and BD, the valve E is kept closed by the external air, the valve B is opened, and a portion of the air which was at first in DB rushes into the Barrel. The external air then forces some water

a little way up the pipe, (to H suppose), until the pressure on the horizontal section of the suction pipe at D, (a pressure which arises from the pressure of the rarified air in HBC together with the pressure of the column of water HD), is equal to the pressure of the external air on an equal area in the surface of the water in the well. When this has taken place, there is equilibrium; and the valve B, being pressed equally on both its surfaces, closes by its own weight.

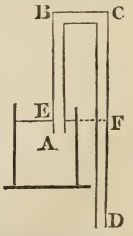
The piston again descends, and the air in AB is driven through E into EF; it ascends again, and more water is forced up the suction pipe: and these operations are repeated until the water rises above B, when on the next descent of the piston the water above B is driven into EF.

As in the case of the common pump, unless BD be less than 34 feet, the pressure of the atmosphere upon the surface of the water in the well will not be enough to raise the water above the piston at B; in which case the machine will not work.

[N. B. The Figure represents the Forcing Pump during the *ascent* of the piston, when the valve E is *shut*, and the air (or water) is rushing from the suction pipe into the Barrel through the *open* valve B.]

30. Prop. xxi. To explain the action of the Siphon.

The Siphon is an instrument for drawing fluids out of vessels. It is a bent pipe or tube, with legs of unequal lengths.



Suppose the Siphon $ABCD$ to be of uniform bore, and the two legs AB and CD parallel and connected together by a pipe BC at right angles to them. Let the instrument, after it is filled with the same fluid as that in the vessel and its ends are stopped, be placed with the legs vertical, and the end A of the *shorter* leg plunged below the surface E of the fluid in the vessel. Let the area of the perpendicular section of the pipe be a square inches, and let F be in the same horizontal line as E. Suppose the ends A and D to be opened, and thin films of fluid to fit the pipe at E and D.

Then, the pressure upwards on the film at D = pressure of the atmosphere on the area a; and the pressure downwards on it = weight of the fluid-column CD.

Similarly, the pressure upwards on the film at E = pressure of the atmosphere on an equal area in the surface of the fluid in the vessel; and the pressure downwards on it = weight of the column BE of fluid.

Wherefore, the whole pressure on the column $EBCD$ of fluid in the direction $EBCD$ is the pressure of the atmosphere on an area a + weight of column CD of fluid; and the whole pressure on the same column in the direction $DCBE$, is the pressure of the atmosphere on an area a + weight of a column BE of fluid*. And since the difference of these is a pressure in the direction $EBCD$ on the column of fluid in the siphon, motion must ensue in that direction, and the fluid must flow out at D. And as fast as the fluid flows out, the pressure of the atmosphere will force up more from the vessel into the siphon, and so the motion will be continued.

[NOTE. If *Water* be the fluid employed, and EB be greater than 34 feet, the pressure of the column BE on the surface a placed at E will be greater than the pressure upwards on that surface which arises from the pressure of the atmosphere on the surface of the water in the vessel. When, therefore, the end A of the siphon is opened, the column BE will begin to sink down until its extremity stands at a height above E in the pipe, such that the weight of the column of water in the tube is just balanced by the pressure of the atmosphere. If D be now opened, the water in CD runs out at D, and, (no water from the vessel flowing over B), the air will rush up the pipe DC and press on the upper end of the column of fluid supported in BE. This column, therefore, being now equally pressed by the atmosphere at each of its extremites, will descend by its own weight into the vessel. THE SIPHON *therefore will not act, when* WATER *is the fluid used, if* EB *be not less than* 34 *feet.*]

* The weight of the fluid in BC produces no pressure on the vertical columns BA and CD, but is supported by the under surface of the pipe BC; the fluid in BC, however, serves to transmit the vertical pressures on AB and DC from the one pipe to the other.

31. PROP. XXII. To graduate a common Thermometer.

THE THERMOMETER is an instrument for comparing the temperatures (i. e. the intensities of the heats) of solids or fluids. [See Figure to Prop. XXIII.]

It consists of a slender tube *BA* of uniform bore, closed at the upper end, and terminating at the lower in a bulb. The bulb and part of the tube are filled with Mercury, Spirits of Wine, or any other fluid (not of a gaseous form) which expands on being heated, and contracts with cold; the remaining part of the tube is a vacuum. If the fluid in the thermometer occupy the same space when the instrument is plunged into two different fluids, the temperature of those fluids is the same.

To graduate a thermometer, plunge the bulb into *melting snow* and make a mark *A* at the point to which the mercury falls in the tube. Next plunge the bulb into *boiling water*, and make another mark *B* at the point to which the mercury rises*. The distance *AB* may then be graduated by equal divisions, and if it be required to compare temperatures by the instrument which are greater than that of boiling water and less than that of melting snow, the graduation may be continued above *B* and below *A*.

32. PROP. XXIII. Having given the number of degrees on *Fahrenheit's* thermometer, to find the corresponding number on the *Centigrade* thermometer.

In *Fahrenheit's* graduation of the scale of the thermometer, (which is that generally made use of in England), 32 is the number placed opposite to the *freezing point A*, and *AB* being divided into 180 parts (*called degrees*), the number placed opposite to the *boiling point B* is 32 + 180, or 212. The graduation of Fahrenheit therefore begins from a point in the scale which is below *A*.

* The temperatures of *melting snow* and of *boiling water* are taken to determine the fixed points *A* and *B* in the scale of the thermometer, because it is found, by experiment, that these temperatures are *fixed* and *invariable;* the mercury *always* falling to *A* when plunged into melting snow, whether the snow be melting quickly or slowly,—and *always* rising to *B* when plunged into boiling water, whether the fire applied to the vessel containing the water be great or small.

In the *Centigrade* thermometer, (which is that generally used on the Continent), the graduation begins from A, and AB is divided into 100 parts, or *degrees*.

Now let there be a thermometer furnished on one side with a scale graduated according to *Fahrenheit*, and on the other with a *Centigrade* scale. Let the mercury rise to any point D, and let F and C be the number of degrees respectively marked opposite to D on the scales of Fahrenheit and the Centigrade.

Then, since the tube is uniform,

The number of degrees of Fahrenheit in AB, (180),

: ..... AD, $(F-32)$,

:: ..... of the Centigrade in AB, (100),

: ..... AD, (C);

$$\therefore\ 180 \times C = 100 \times (F-32);$$

$$\therefore\ C = \frac{100}{180} \cdot (F-32) = \frac{5}{9} \cdot (F-32).$$

Ex. If the mercury in Fahrenheit's thermometer stand at 77, then the corresponding number of degrees on the Centigrade will be

$$\frac{5}{9} \cdot (77-32) = \frac{5}{9} \cdot 45 = 25.$$

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The Barometer.

The Air-Pump.

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